discrete symmetries and helicity

Parity (P) $\vec{x} \to -\vec{x}$, $\vec{p} \to -\vec{p}$, $\vec{S} \to \vec{S}$ $(\vec{x} \times \vec{p})$

$\Rightarrow h \to -h$

$\Rightarrow$ a theory with only $+\frac{1}{2}$ helicity cannot be parity invariant. weak interactions in SM are not parity inv.!

time reversal (T) $t \to -t$, $\vec{x} \to \vec{x}$, $\vec{p} \to -\vec{p}$, $\vec{S} \to -\vec{S}$

$\Rightarrow h \to h$

charge conjugation (C): $(t, \vec{x}) \to (t, \vec{x})$, $h \to h$

particle $\to$ anti-particle (e.g. $e^- \to e^+$)

can prove CPT theorem: CPT is always a symmetry in local Lorentz-invariant QFT

CPT: $(t, \vec{x}) \to (-t, -\vec{x})$, particle $\to$ anti particle

$E, \vec{p} \to E, \vec{p}$, $\vec{S} \to -\vec{S}$, $h \to -h$

$\Rightarrow$ massless $h = \frac{1}{2}$ fermion must $h = -\frac{1}{2}$ antifermion

Spin 1 (e.g. photon γ)

$$\hat{h} = \hat{p} \cdot \vec{S}$$

three possible polarizations $\vec{\epsilon}^{\,\hat{p}}_{\substack{\pm \\ 0}}$: $\quad \hat{h}\,\vec{\epsilon}^{\,\hat{p}}_{\pm} = \pm\hbar\,\vec{\epsilon}^{\,\hat{p}}_{\pm}$

$$\hat{h}\,\vec{\epsilon}^{\,\hat{p}}_{0} = 0$$

"+" right-handed polarization
"−" left- " "
"0" longitudinal

note that $\hat{p} \cdot \vec{\epsilon}^{\,\hat{p}}_{\pm} = 0$ "transverse"

$\hat{p} \cdot \vec{\epsilon}^{\,\hat{p}}_{0} = 1$ "longitudinal"

h is again Lorentz-invariant $\Rightarrow$ can have just $h = +\hbar$ (or − or 0)

but (CPT): a theory with a $+\hbar$ photon must also have a $-\hbar$ "anti-photon". But photons are their own antiparticles (no charge) $\Rightarrow$ the photon has $\pm\hbar$ helicities but no longitudinal polarization.

Note: will often look at scattering of particles with $\frac{m}{E} \ll 1 \Rightarrow m = 0$ is a good approximation

particles & fields

Relativity + Lorentz invariance : signals $v \leq c$

Locality

signals travel via wiggles in fields, wave equation

speed of waves $v \leq c$

QM : quantize fields by treating field oscillations as QM harmonic oscillators

oscillator excitations $\longleftrightarrow$ particles

free particle states in mom. space

$|0\rangle$ vacuum

$|\vec{p}\rangle$ 1 particle of momentum p

$|\vec{p}, \vec{p}'\rangle$ 2 particles ...

$\vdots$

$\underbrace{\qquad\qquad}$

Fock space

creation operators $a^+_{\vec{p}}|0\rangle = |\vec{p}\rangle$

$a^+_{\vec{p}}\, a^+_{\vec{p}'}|0\rangle = |\vec{p}, \vec{p}'\rangle$ ...

destruction operators $a_{\vec{p}}$ hermitian conjugate $\langle \vec{p}|a^+_{\vec{p}}|0\rangle \sim 1$

$\langle\;\rangle^* = \langle 0|a_{\vec{p}}|\vec{p}\rangle \sim 1$

real field operator creating particle in position space

$$\phi(x) \sim \int d^3p \; e^{i\vec{p}\cdot\vec{x}} \, a^+_{\vec{p}} + e^{-i\vec{p}\cdot\vec{x}} \, a_{\vec{p}}$$

$(\phi = \phi^+)$

create particle — destroy particle

complex field

$$\phi \sim e^{i\vec{p}\cdot\vec{x}} \, a^+_{\vec{p}} + e^{-i\vec{p}\cdot\vec{x}} \, b_{\vec{p}}$$

charge q — destroy anti-particle Charge $-q$

$$\phi^+ \sim b^+ + a$$

2 properties of QFT

1. antiparticles: ϕ creates particles h q

   destroys anti-particles $-h, -q$

2. spin-statistics: integer spins are bosons, commute at space-like separation $[\phi(x), \phi(y)] = 0$

   half-integer spins are fermions

   $\{\psi(x), \psi(y)\} = 0$ (some - signs)

Relativistic normalization of states

non-relativistic: $\langle \vec{p} \,|\, \vec{p}\,' \rangle = (2\pi)^3 \, \delta^3(\vec{p} - \vec{p}\,')$

convention

generalization of δ_{ij} to continuous

"completeness"

$\Longleftrightarrow$ $\mathbb{1} = \int \frac{d^3p}{(2\pi)^3} |\vec{p}\rangle\langle\vec{p}|$ so that $\mathbb{1}|\vec{p}\,'\rangle = |\vec{p}\,'\rangle$

rotationally invariant

proof: change of basis $\vec{p}\,' = \mathcal{O}\vec{p}$

rotation matrix

"orthogonal"

$$d^3p \to d^3p' = d^3p \, |\det(\mathcal{O})| = d^3p \quad \blacksquare$$

"Jacobian"

but not boost invariant. $\Rightarrow$ normalization of states is not boost invariant.

BUT $\int \frac{d^4p}{(2\pi)^4}$ is rotation and boost invariant

$$d^4p' = d^4p \, |\det \Lambda| = d^4p$$

Lorentz transformation

consider: $$\int \frac{d^4p}{(2\pi)^4}\, 2\pi\, \delta_+(p^2 - m^2)$$ Lorentz invariant

$$\delta_+\left(p_0^2 - (\vec{p}^{\,2} + m^2)\right) = \frac{\delta\left(p_0 - \overbrace{\sqrt{\vec{p}^{\,2} + m^2}}^{E_p}\right)}{2E_p}$$

"pick positive root"

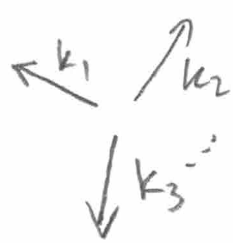
using $$\delta(f(x)) = \frac{\delta(x - x_0)}{|f'(x_0)|}$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p}$$ Lorentz invariant $\overbrace{\text{measure}}^{\text{integration}}$

$\Rightarrow$ define: $$\boxed{\begin{aligned} \mathbb{1} &= \int \frac{d^3p}{(2\pi)^3\, 2E_p} |\vec{p}\rangle\langle\vec{p}| \\ \Leftrightarrow \quad \langle \vec{p} | \vec{p}\,' \rangle &= (2\pi)^3\, 2E_p\, \delta^3(\vec{p} - \vec{p}\,') \end{aligned}}$$

<u>Scatterings</u>

in $\xrightarrow{p_1} \quad \xleftarrow{p_2}$ $\qquad |p_1 p_2\rangle$

out 

$\langle k_1 \ldots k_n|$