PY 751 particle physics the Standard Model (SM)

will use "natural" units $\hbar = c = 1$

fundamental particle properties

	spin	charge	baryon#	lepton#	mass
$u\ c\ t$	$1/2$	$2/3$	$1/3$	0	~few MeV, ~1.2 GeV, 175 GeV
$d\ s\ b$	$1/2$	$-1/3$	$1/3$	0	$> m_u$, ~100 MeV, ~4 GeV
$e\ \mu\ \tau$	$1/2$	-1	0	1	0.5 MeV, 105 MeV, 1.78 GeV
$\nu_e\ \nu_\mu\ \nu_\tau$	$1/2$	0	0	1	$\lesssim 0.1$ eV
γ	1	0	0	0	0
$W^\pm\ Z$	1	$\pm1, 0$	0	0	~80 GeV, ~91 GeV
g	1	0	0	0	0
graviton	2	0	0	0	0
h	0	0	0	0	125 GeV

- spin ½ are fermions, called "matter fields"
- γ W, Z, g are "gauge bosons", spin 1, force carriers
- h "higgs particle", quantum of Higgs field which gives masses through Higgs mechanism

strong interaction bound states "hadrons"

the strong interactions confine colored particles into color-neutral bound states. The lightest ones are the most stable → most important.

mesons:

		valence quarks	charge	B	L	mass
pion	$\pi^\pm$	$u\bar{d}$, $\bar{u}d$	± 1	0	0	~140 MeV
	π^0	$\bar{u}u - \bar{d}d$	0	0	0	~135 MeV

decays: $\pi^+ \to \mu^+ \nu_\mu$, $e^+ \nu_e$ — weak decays, long lifetime on detector time scales

$\pi^0 \to \gamma\gamma$ — electromagnetic "prompt"

baryons:

p^+	uud	1	1	0	938.3 MeV
n^0	udd	0	1	0	939.6 MeV

decays: $n^0 \to p^+ e^- \bar{\nu}_e$ — minutes

p^+ stable? (stable in SM)

the SM (gauge) interactions

electromagnetic

- long-range

strong

- short range — scales: $\Lambda \sim$ GeV binding energy
  - $\hbar/\Lambda$ time scale
  - $\hbar c/\Lambda$ distance scale
- confines colored particles into color neutral bound states
- "MIT bag model": (u u ū u d g) p

  "binding energy of bag" $\sim$ GeV $\sim \Lambda$

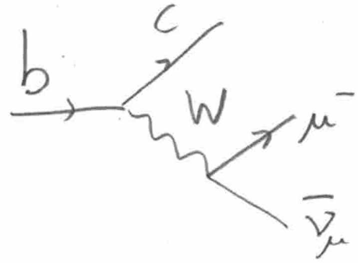
  size of bag $\hbar c/\Lambda$

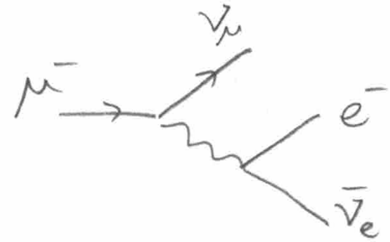
weak

- very short range $\hbar c/m_W \sim \hbar c/100\,\text{GeV}$
- radioactive decays

  flavor changing (strong, E&M both preserve flavor)

  e.g.





Relativistic kinematics:

Einstein $\quad E^2 = m^2c^4 + \vec{p}^{\,2}c^2 \qquad$ set $c=1$

using 4-vectors: $\quad p^{\mu} = (E, \vec{p})$ ← greek

$$p^2 \equiv \sum_{\mu=0}^{3} p_\mu p^\mu \equiv \underbrace{p_\mu p^\mu}_{\text{summation convention}} = \eta_{\mu\nu}\, p^\mu p^\nu = E^2 - \vec{p}^{\,2}$$

$$\eta = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$\Rightarrow$ Einstein: $\quad p^2 = m^2$

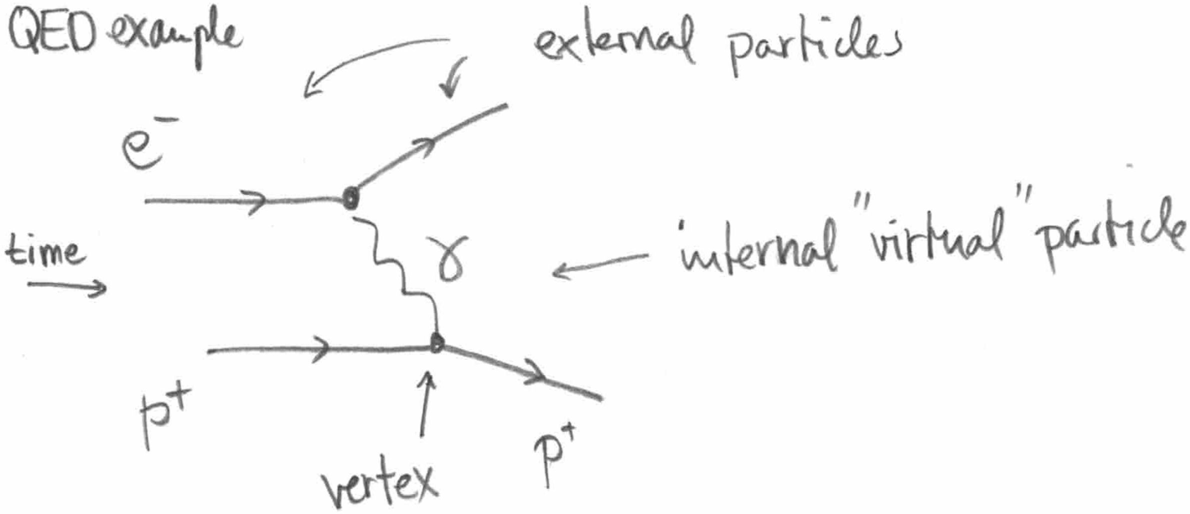
massless particle (photon): $\quad m^2 = 0 \Rightarrow E = |\vec{p}|$

$$\Rightarrow \quad p^\mu \equiv (E, \vec{p}) = (\hbar\omega, \vec{p}) = (c|\vec{p}|, \vec{p}) = |\vec{p}|(1, \hat{p})$$

for the most part interactions can be treated as perturbations. We will expand about free particle states and add interactions as perturbations. $\Rightarrow$ need to study free particle states first.

Feynman diagrams are representations of scattering amplitudes in perturbation theory $\langle \text{final} \mid \text{initial} \rangle$

QED example



- external particles are free particles "on-shell" and satisfy
  $p^2 = m^2$
- internal particles are off-shell $p^2 \neq m^2$
- overall 4-momentum conservation is ensured by 4-mom conservation at each vertex

  e.g. $P_i^{e^-} = P_f^{e^-} + P_\gamma$

Spin

Case 1: $m \neq 0$ want a relativistic theory (i.e. boost + rotation invariance)

$\Rightarrow$ can go to the rest frame

$\underset{m}{\bullet}$ $\leftarrow$ rotationally invariant except for possible spin

example spin ½ (electron)

spin wave function = 2-component "spinors" ξ

spin operator $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$ ← pauli

can choose eigenstates of S_z as basis: $S_z \xi_\pm = \pm \frac{\hbar}{2}\xi_\pm$

$\xi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ $\xi_- = \begin{pmatrix}0\\1\end{pmatrix}$ (normalized $|\xi|^2 = 1$)

$\Rightarrow$ a massive spin ½ particle has 2 possible independent polarization states $\uparrow$ or $\downarrow$, note that they are related by a rotation by 180°.

$\Rightarrow$ a rotationally invariant theory must allow both states.

probability to measure spin ½ in +x direction after preparing particle in ξ^z_+ state?

$$P = |A|^2 \equiv \left| \langle \xi^x_+ | \xi^z_+ \rangle \right|^2 = \left| (\xi^x_+)^\dagger \xi^z_+ \right|^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = 1/2$$

using: ξ^x_+ is eigenspinor of $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ with $\sigma_x \xi^x_+ = +\xi^x_+$

$$\Rightarrow \quad \xi^x_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

<u>example spin 1</u> Z-boson

$$\vec{S} = \hbar \vec{J}, \quad J_1 = \begin{pmatrix} 0 & & \\ & & -i \\ & i & \end{pmatrix} \quad J_2 = \begin{pmatrix} & & i \\ & 0 & \\ -i & & \end{pmatrix} \quad J_3 = \begin{pmatrix} & -i & \\ i & & \\ & & 0 \end{pmatrix}$$

Rotations are $R(\vec{\theta}) = e^{-i \vec{\theta} \cdot \vec{J}}$

generators of rotations

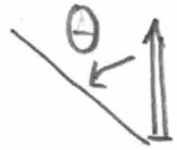
rotation axis $\hat{\theta}$, angle $|\vec{\theta}|$

quantize spin along z-axis:

$$\left.\begin{array}{ll} \epsilon_\pm = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix} & s_z = \pm \hbar \\ \epsilon_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} & s_z = 0 \end{array}\right\} \text{3 independent polarizations}$$

probability to measure $+\hbar$ in θ direction if prepare in $\hat{z}$ direction?

$$\left|\langle \epsilon_+^\theta | \epsilon_+^z \rangle\right|^2 = P$$



↑

need ϵ_+^θ, trick: rotate ϵ_+^z by angle θ about y-axis (or z-axis)

$$\epsilon_+^\theta = R_y(\theta)\,\epsilon_+^z = e^{-i\theta J_2}\,\epsilon_+^z = \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} c_\theta \\ i \\ -s_\theta \end{pmatrix}$$

$$\Rightarrow P = \left|\left(\epsilon_+^\theta\right)^\dagger \epsilon_+\right|^2 = \frac{1}{4}\left|\begin{pmatrix} c_\theta \\ -i \\ -s_\theta \end{pmatrix} \cdot \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}\right|^2 = \frac{1}{4}(1+c_\theta)^2$$

note $P = \begin{cases} 1 & \text{for } \theta = 0 \\ 0 & \theta = \pi \end{cases}$ ✓

$\underline{m=0}$ no rest frame $\Longrightarrow$ preferred direction $\hat{p}$

$\Rightarrow$ quantize spin along $\hat{p}$: $\hat{h} = \hat{p} \cdot \vec{S}$ = "helicity" operator

e.g. massless e^- :

$h = +1/2$

helicity eigenvalue, i.e. helicity of the electron

helicity is preserved under rotations

$h = +1/2$

and boosts

(cannot overtake a particle traveling at speed of light)

(massive case:

$h = +1/2$ $\qquad$ $h = -1/2$)

$\Rightarrow$ a Lorentz invariant theory is consistent with having only + (or −) helicity massless particles, it need not have both!