# Cosmological Physics PY 555

History of the Universe, quantitative science

historical surprises:

- 70's dark matter (vera Rubin) — non-relativistic particles
  ~ 5 times energy density of visible matter

- '98 dark energy (Riess et al) — expansion rate is accelerating ⇒ vacuum energy density ~ 70% of total today
  ΛCDM model

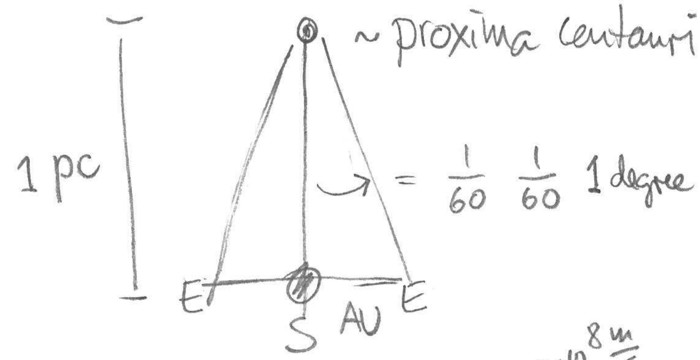
- 80s inflation (Guth) — quantum fluctuations during inflation are initial conditions to structure formation
  (Pi)

# Scales

parsec

pc

nearest star



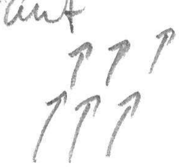

| | | |
|---|---|---|
| Earth orbit | $10^{-5}$ pc | |
| nearest star | pc | $\simeq 3.26$ ly $= c \cdot yr$ |
| to MW center | 8 kpc | |
| MW | 30 kpc | |
| DM halo of MW | 0.1 Mpc | |
| Galaxy clusters | 1-10 Mpc | |
| superclusters | 100 Mpc | |
| visible universe | 3000 Mpc | |

$c = 3 \cdot 10^8 \frac{m}{s}$, $yr \approx \pi\ 10^7 s$

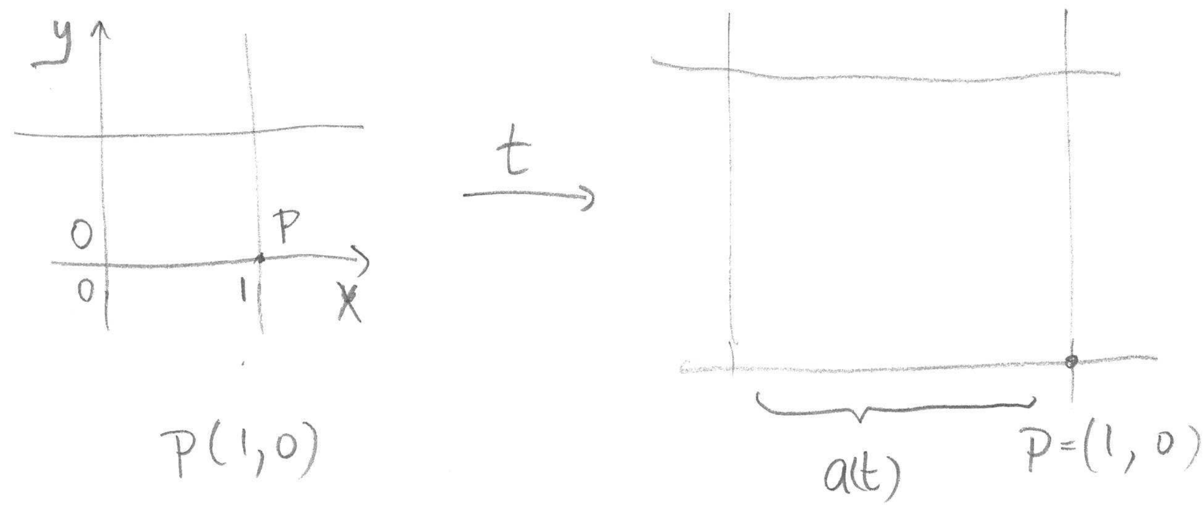
at large scales $\gtrsim 10$ Mpc universe is

- homogeneous — translation invariant
- isotropic — rot. invariant

Q: What's homog. & not isotr.? A.



## expanding universe




"comoving coordinates"

distance in co-moving coordinates stays fixed $\overline{OP} = 1$

" " physical " $\Delta \ell = 1 \cdot a(t)$

physical distance defined with "metric"

$$d\ell^2 = (dx\ dy)\begin{pmatrix} a^2(t) & 0 \\ 0 & a^2(t) \end{pmatrix}\begin{pmatrix} dx \\ dy \end{pmatrix} = a^2(dx^2+dy^2)$$

NB. motion relative to co-moving coordinates is called "peculiar" motion

<u>example</u> metric for polar coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$dx = dr\, c - d\theta\, rs \qquad (dx = dr\, c + r\, dc)$$
$$dy = dr\, s + d\theta\, rc$$

$$dl^2 = dx^2 + dy^2 = dr^2(c^2+s^2) + d\theta^2 r^2 (c^2+s^2) + dr\, d\theta \cdot 0$$
$$= dr^2 + r^2 d\theta^2 = (dr \;\; d\theta) \underbrace{\begin{pmatrix} 1 & \\ & r^2 \end{pmatrix}}_{\text{metric in p.c.}} \begin{pmatrix} dr \\ d\theta \end{pmatrix}$$

$dx^2 + dy^2$ manifestly translation invariant
$dr^2 + r^2 d\theta^2$ manifestly rot. invariant

$\Rightarrow dl^2$ is transl. + rot. invariant homog + isotropic

## 3D spherical

$$d\ell^2 = dr^2 + r^2 d\Omega^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2$$

3d rotation invariant $\Rightarrow$ choose same units for $x, y, z$

$d\ell^2$ is physical length $\Rightarrow$ coordinate independent

## 4D space-time

$$x^\mu = (t, \vec{r}) \qquad \left[\, p^\mu = (E, \vec{p}) \,\right]$$

Special relativity: physical laws are invariant under $\underbrace{\text{rotations \& boosts}}_{\text{Lorentz transformations}}$

$\Rightarrow$ choose same units for $t, x, y, z$

Rotations
$$\left(\begin{array}{c|cc c} 1 & & & \\ \hline & c & s & \\ & -s & c & \\ & & & 1 \end{array}\right)$$
rot about z-axis

$\cos\theta$

Boosts
$$\left(\begin{array}{cc|cc} ch & sh & & \\ sh & ch & & \\ \hline & & 1 & \\ & & & 1 \end{array}\right)$$

$\cosh \xi$, $\tanh \xi = \frac{v}{c}$

$\uparrow$ rapidity

4D line element

$$ds^2 = (dt \ d\vec{x}) \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \begin{pmatrix} dt \\ d\vec{x} \end{pmatrix}$$

$$= dt^2 - \sum_i dx_i^2 = dt^2 - d\vec{x}^2$$ in Minkowski space and cartesian coord's.

$$= g_{\mu\nu} dx^\mu dx^\nu$$ in general coordinates & space

summation convention

GR: $g_{\mu\nu} = g_{\mu\nu}(x^\alpha)$

- metric depends on space & time
- encodes gravity
- responds to Energy-Momentum

What is the metric of the universe?

homog. + isotr. $\Rightarrow \exists$ time slices s/th

$$g = \left( \begin{array}{c|c} b^2(t) & \vec{0} \\ \hline \vec{0} & -a^2(t)\, dl^2 \end{array} \right)$$

vector, if $\neq 0$, not isotropic

3d symmetric

redefine $dt' = b(t)\,dt$ to remove $b(t)$

$$\Rightarrow \quad g = \begin{pmatrix} 1 & & & \\ & -a^2 & & \\ & & -a^2 & \\ & & & -a^2 \end{pmatrix}$$

$$ds^2 = dt^2 - a^2(t)\underbrace{(dx^2+dy^2+dz^2)}_{dl^2}$$ FRW metric w. flat 3d slices

$$= dt^2 - a^2(t)\left(dr^2 + r^2 d\Omega^2\right)$$ spherical

<u>too restrictive</u>: could have allowed <u>curvature</u> (3d slices with)

<u>example</u>: 2d space with curvature $>0$

$S^2$ surface of sphere - 2d space

$x^2+y^2+z^2 = u^2$

translation invariant, rot. inv.



$\alpha+\beta+\gamma > 180°$

curvature

metric? $dl^2 = dx^2 + dy^2 + dz^2$

eliminate $z$ and use polar coord for $x, y$

$r^2 + z^2 = u^2$

$\Rightarrow r\,dr + z\,dz = 0 \Rightarrow dz^2 = \frac{r^2 dr^2}{z^2} = \frac{r^2 dr^2}{u^2 - r^2}$

$\Rightarrow dl^2 = dr^2 + r^2 d\theta^2 + r^2 \frac{dr^2}{u^2 - r^2} = \frac{dr^2}{1 - r^2/u^2} + r^2 d\theta^2$

rescale: $\underset{\text{length}}{r} = u \underset{\text{dimensionless}}{r'}$

$\Rightarrow u^2 \left[ \frac{dr'^2}{1 - r'^2} + r'^2 d\theta^2 \right]$

absorb into $a(t)$

$r' = 1$ coordinate singularity but not singularity on $S^2$

Hyperbolic space $H^2$ $r^2 - z^2 = -u^2$

$dl^2 = dr^2 - dz^2$

$\Rightarrow \frac{dr'^2}{1 + r'^2} + r'^2 d\theta^2$

1/28/19