Deconfined quantum criticality

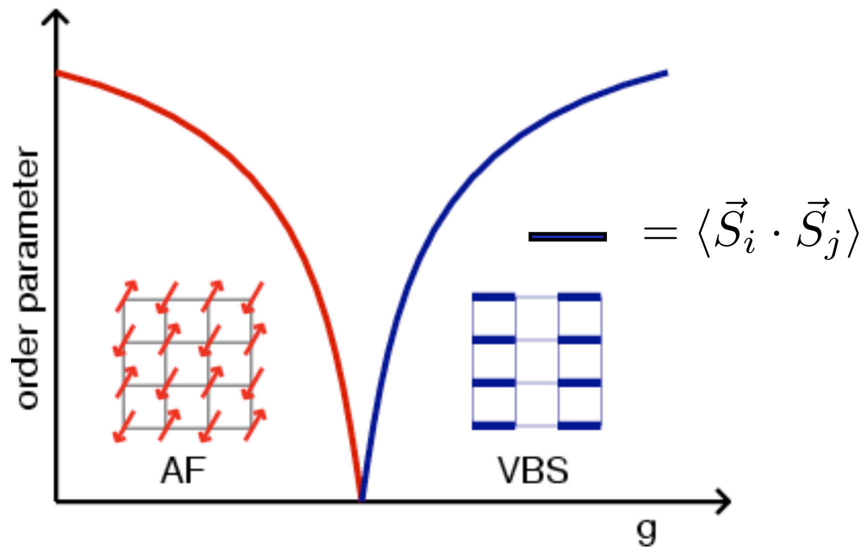
[Senthil et al., Science 303, 1490 (2004)]

$$\mathbf{H} = \mathbf{J} \sum_{\langle \mathbf{i},\mathbf{j} \rangle} \mathbf{S_i} \cdot \mathbf{S_j} + \mathbf{g} \times \cdots$$

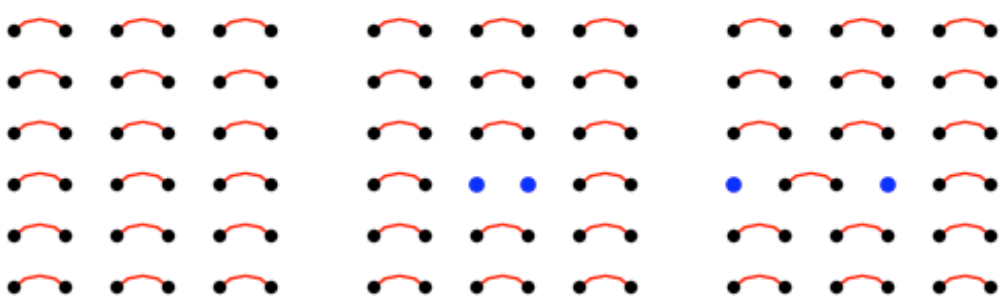
Quantum phase transition in a 2D system with one spin per unit cell
- antiferromagnetic for small g
- valence-bond solid (VBS) for large g (spontaneously broken symmetry)

Questions

- is the transition continuous?
  - normally order-order transitions are first order (Landau-Ginzburg)
  - theory of deconfined quantum critical points has continuous transition
- nature of the VBS fluctuations?
  - emergent U(1) symmetry predicted



Spinon deconfinement upon approaching the critical point



Confinement inside VBS phase associated with new length scale and emergent U(1) symmetry

How can we study deconfined quantum-criticality in a model system?

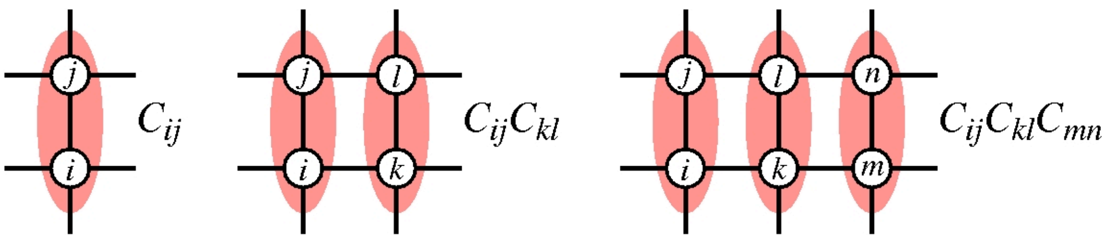
- the theory is based on continuum field-theory (Lagrangian)
- is there a reasonably microscopic model (Hamiltonian) with this physics?
- frustrated models (e.g., J_1-J_2 Heisenberg) are good candidates
  - but no large-scale simulations (QMC) are possible
- **Look for non-frustrated models with Néel - VBS transition**

The Heisenberg interaction is equivalent to a singlet-projector

$$C_{ij} = \frac{1}{4} - \vec{S}_i \cdot \vec{S}_j$$

$$C_{ij}|\phi^s_{ij}\rangle = |\phi^s_{ij}\rangle, \qquad C_{ij}|\phi^{tm}_{ij}\rangle = 0 \quad (m = -1, 0, 1)$$

- we can construct models with products of singlet projectors
- no frustration in the conventional sense (QMC can be used)
- multiple-singlet projection reduces the antiferromagnetic order



The original "J-Q" model is [Sandvik, PRL 2007]

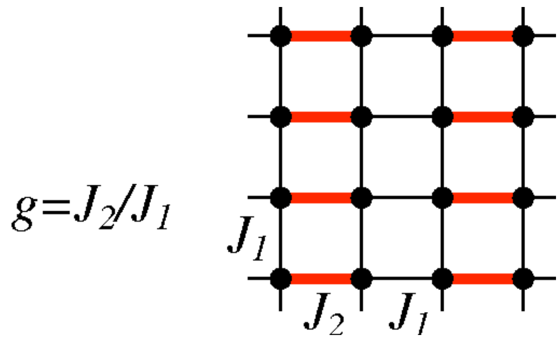
$$H = -J \sum_{\langle ij \rangle} C_{ij} - Q \sum_{\langle ijkl \rangle} C_{ij} C_{kl}$$

- Has Néel-VBS transition, appears to be continuous (more later, with QMC)

Finite-lattice calculations

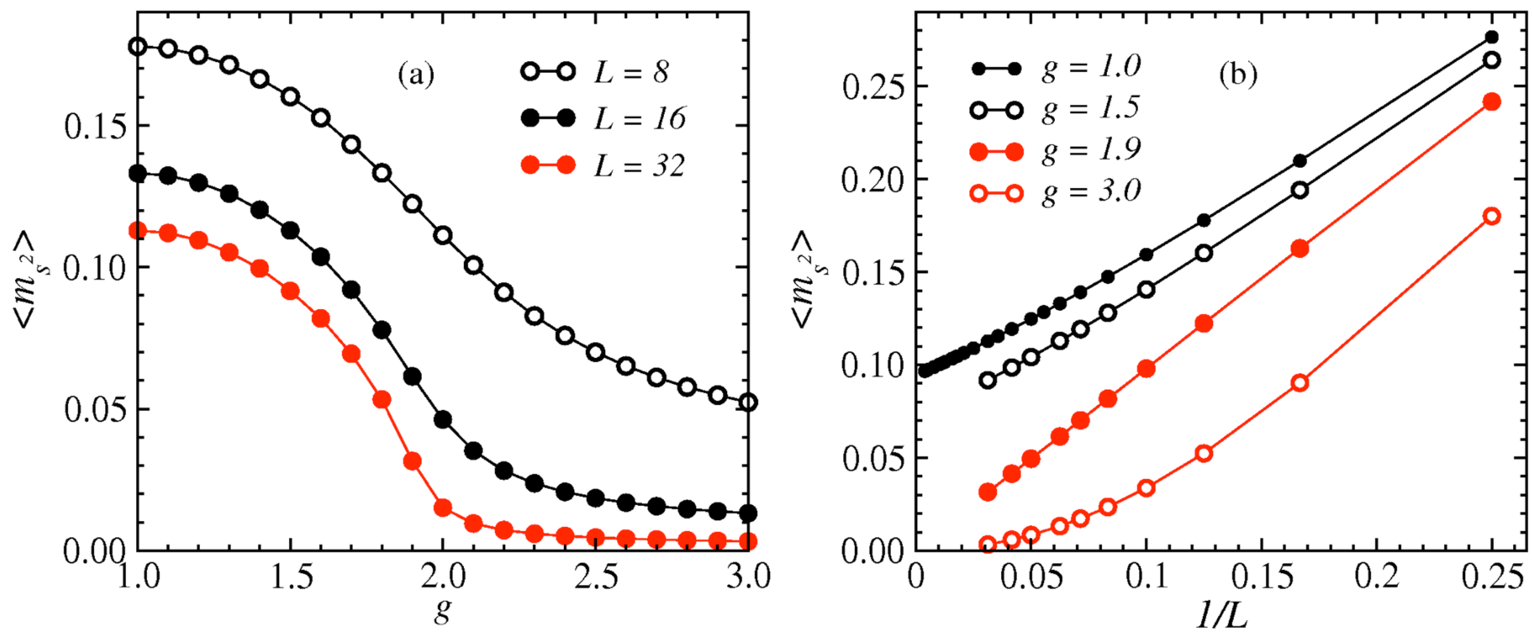
We will do numerically exact calculations (no approximations) for finite lattices

- extrapolate to infinite size, to learn about
  - the ground state and excitations
  - nature of quantum phase transitions
  - associated T>0 physics



Example: Dimerized Heisenberg model

- QMC results for L×L lattices



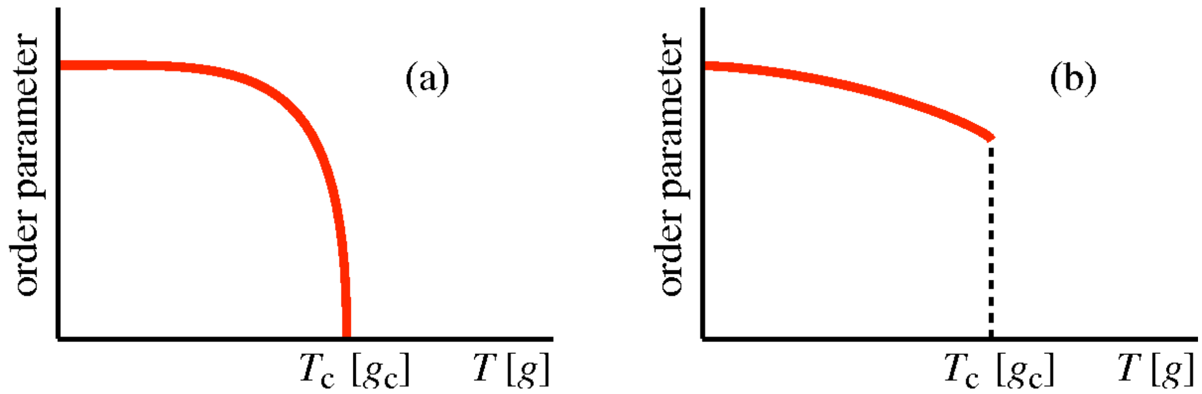
It is often known how various quantities should depend on L

- in a Néel state, spin-wave theory → $\langle m_s^2(L)\rangle = \langle m_s^2(\infty)\rangle + a/L + \ldots$
- use finite-size scaling theory to study the quantum-critical point

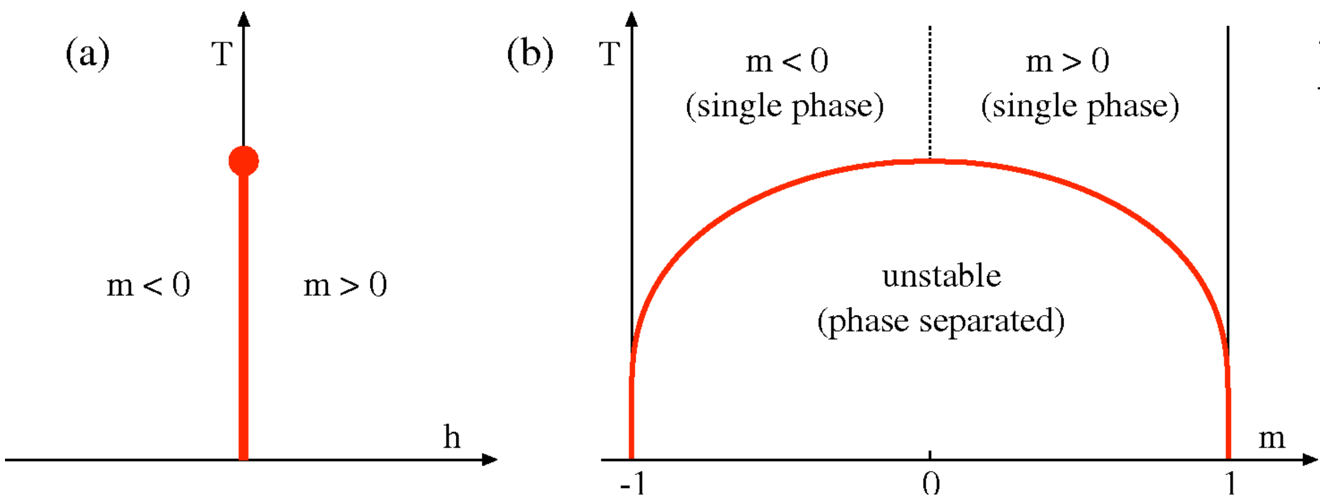
Finite-size scaling and critical points

Phase transitions can be continuous or first-order (discontinuous)

- occur as a function of T or some parameter g at T=0
- we want to analyze, especially, continuous transitions



Prototypical classical model with a critical point: 2D Ising model

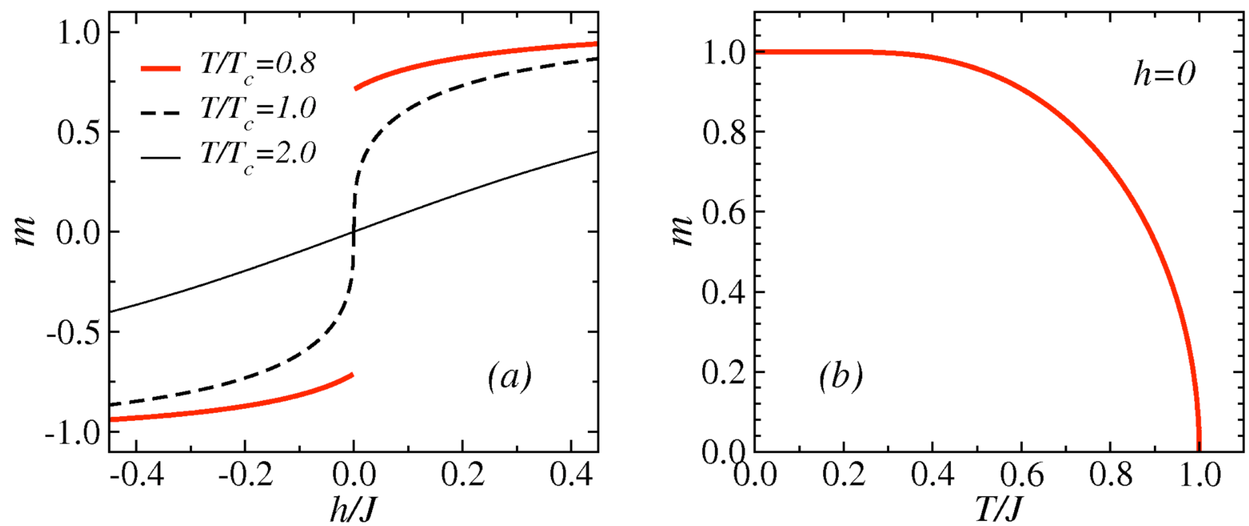


$$\frac{T_c}{J} = \frac{2}{\ln(1+\sqrt{2})}$$

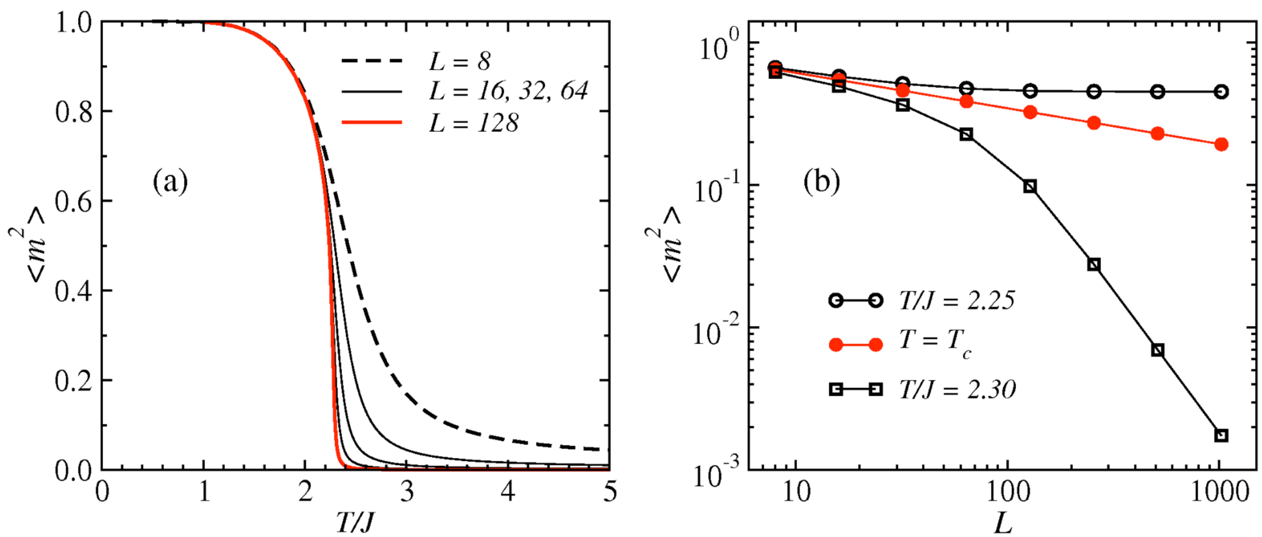
$$\approx 2.269$$

- first-order transition versus h (at h=0) for $T<T_c$
- continuous transition at h=0

Mean-field solution: $J = J_i = \sum_j J_{ij} \qquad m = \tanh[(Jm + h)/T], \qquad (m = \langle \sigma_i \rangle)$



Monte Carlo results for L×L lattices (cluster algorithm, very small error bars)



Phase transition becomes apparent for large L

Critical exponent β can be extracted from L-scaling at Tc

$$\langle m(\infty, T) \rangle \sim |T - T_c|^{\beta}$$

$$\langle m^2(L, T_c) \rangle \sim L^{-2\beta/\nu}$$

Review of criticality and scaling

We discuss Ising spins for simplicity (but most results generic)

Correlation function: $C(\mathbf{r}_{ij}) = \langle \sigma_i \sigma_j \rangle$

For large r:

$$C(r) \to \begin{cases} e^{-r/\xi} & T > T_c \\ r^{-(d-2+\eta)} & T = T_c \\ \langle m^2 \rangle & T < T_c \end{cases}$$

"Connected" correlation function for $T<T_c$:

$$\bar{C}(r) = C(r) - \langle |m| \rangle^2 \to e^{-r/\xi}$$

Note that the magnetization can be computed in a finite system in several ways, all equal when N→∞:

$$\langle |m| \rangle, \quad \sqrt{\langle m^2 \rangle}, \quad \sqrt{C(r_{\max})}, \qquad \left[m = \frac{1}{N} \sum_{i=1}^{N} \sigma_i \right]$$

The squared magnetization can be exactly written in terms of C(r):

$$\langle m^2 \rangle = \frac{1}{N} \sum_{\mathbf{r}} C(\mathbf{r})$$

For N→∞ close to T_c, $T<T_c$:

$$\beta = \begin{cases} 1/8, & \text{2D Ising (exact)} \\ 1/2, & \text{mean-field (generic)} \\ ..., & \text{other universality classes} \end{cases}$$

$$\langle m \rangle \sim |t|^{\beta}, \qquad t = (T - T_c)/T_c$$

The correlation length diverges as $T \to T_c$ (from above and below)

$$\xi \sim |t|^{-\nu}$$

2D Ising: ν=1, mean-field: ν=1/2,...

Susceptibility (linear response function)

$$\chi = \frac{d\langle m\rangle}{dh}\Big|_{h\to 0} = \frac{N}{T}\left(\langle m^2\rangle - \langle |m|\rangle^2\right)$$

also diverges as $T \to T_c$

$$\chi \sim |t|^{-\gamma}$$

Finite-size scaling

How are divergencies (and other singularities) affected by finite N?

Consider some quantity which has the form (N=∞)

$$Q \sim |t|^{-\kappa}$$

We can write |t| as: $|t| \sim \xi^{-1/\nu}$, which gives

$$Q \sim \xi^{\kappa/\nu}$$

The maximum correlation length is L, at $T_c(L)$. Substitute **$\xi \to L$**

$$Q[T_c(L)] \sim L^{\kappa/\nu} \qquad |t|_L = [T_c(L) - T_c(\infty)]/T_c(\infty) \sim L^{-1/\nu}$$

More general finite-size scaling hypothesis

- has been justified using the renormalization-group theory

$$Q(t,L) = L^{\sigma} f(\xi/L),$$

Using $\xi \sim |t|^{-1/\nu} \to$

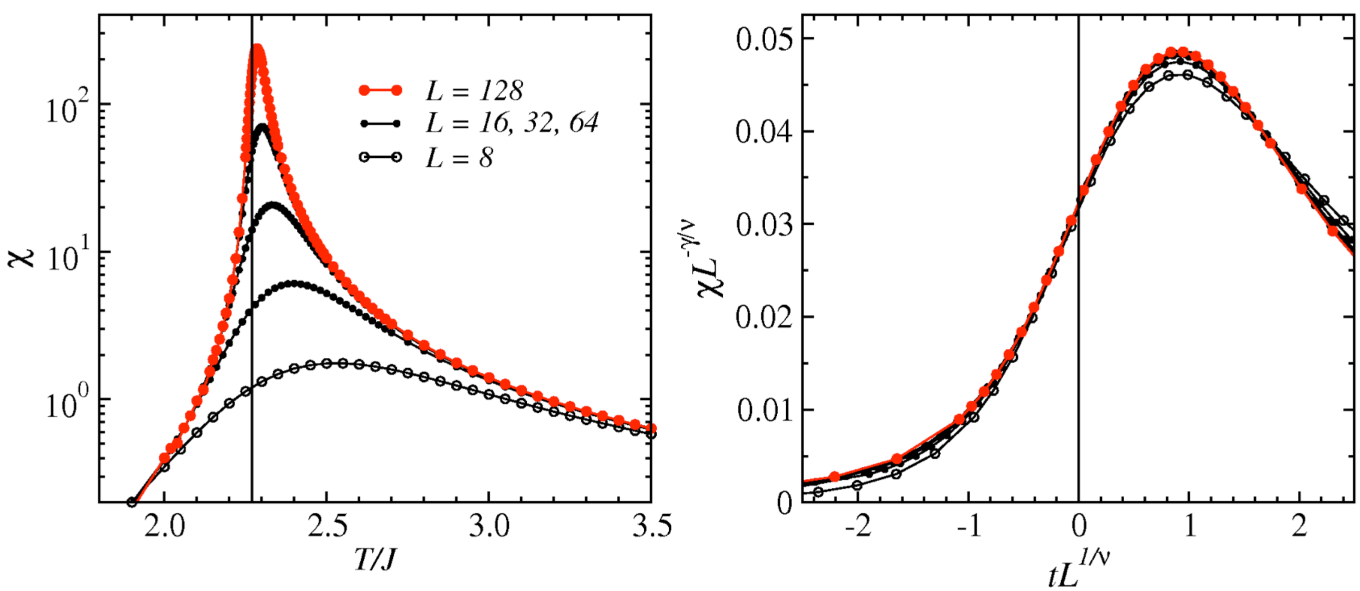
$$Q(t,L) = L^{\sigma} g(tL^{1/\nu})$$

From this we must be able to reproduce infinite-size form:

$$Q(t, L \to \infty) \sim |t|^{-\kappa}$$

which is the case if $g(x) \sim x^{-\kappa}$ and $\sigma = \kappa/\nu$

Test: susceptibility of 2D Ising model (Monte Carlo)



$T_c = 2/\ln(1+\sqrt{2})$

$\nu = 1, \gamma = 7/4$

Normally: adjust Tc and exponents so that the data "collapse"