Smoothing Rocks by Chipping

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PRE **75**, xxxx (2007), cond-mat/0611415

NSF grants CHE0532969 (PLK) and DMR0535503 (SR).

2007 APS March Meeting

Basic question:

inspired by Durian et al., PRL 97, 028001 (2006); PRE 75, 021301 (2007)

What is the shape of rocks as they erode?

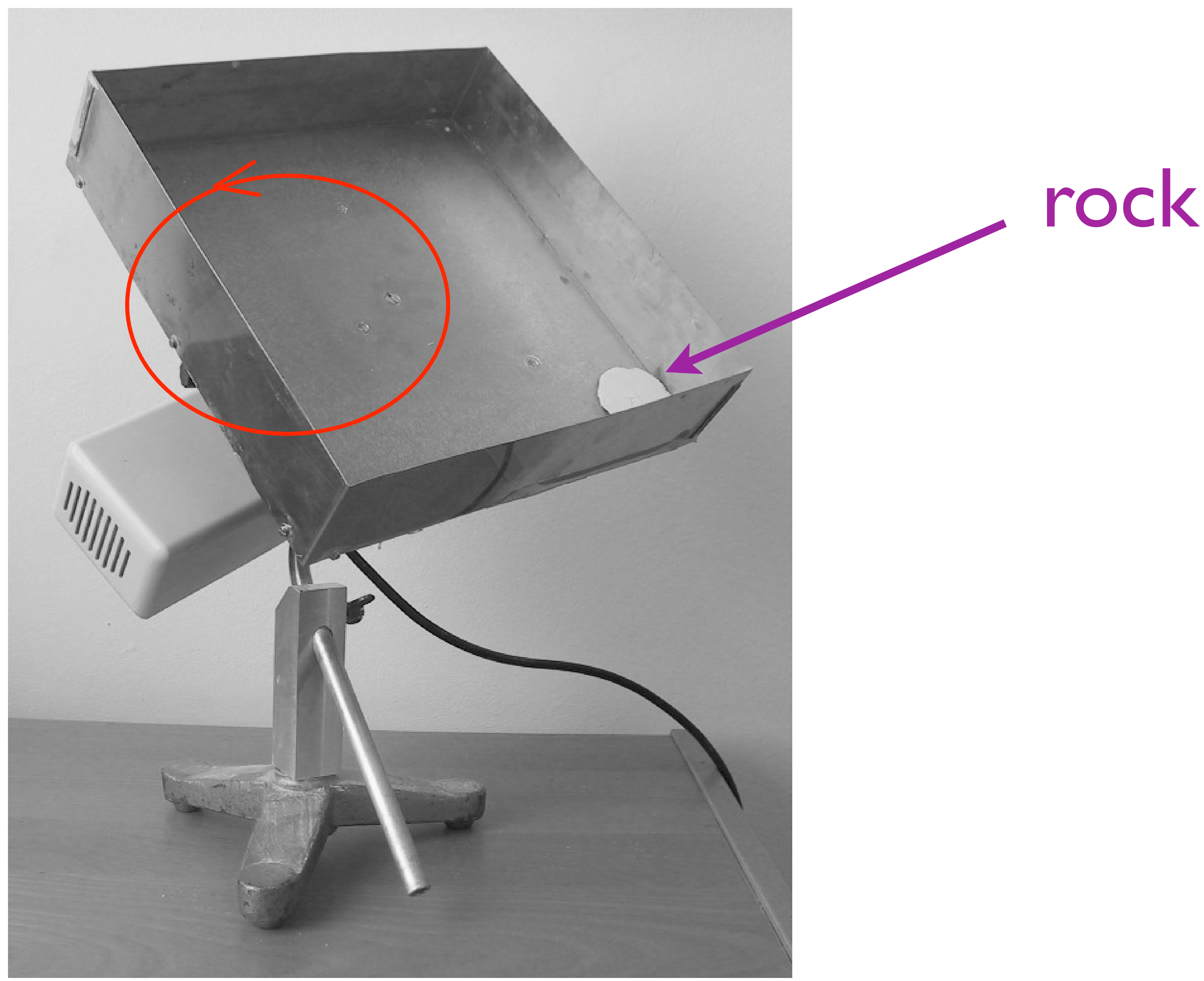
Aristotle:

Rounding by faster erosion at exposed corners.

Main result:

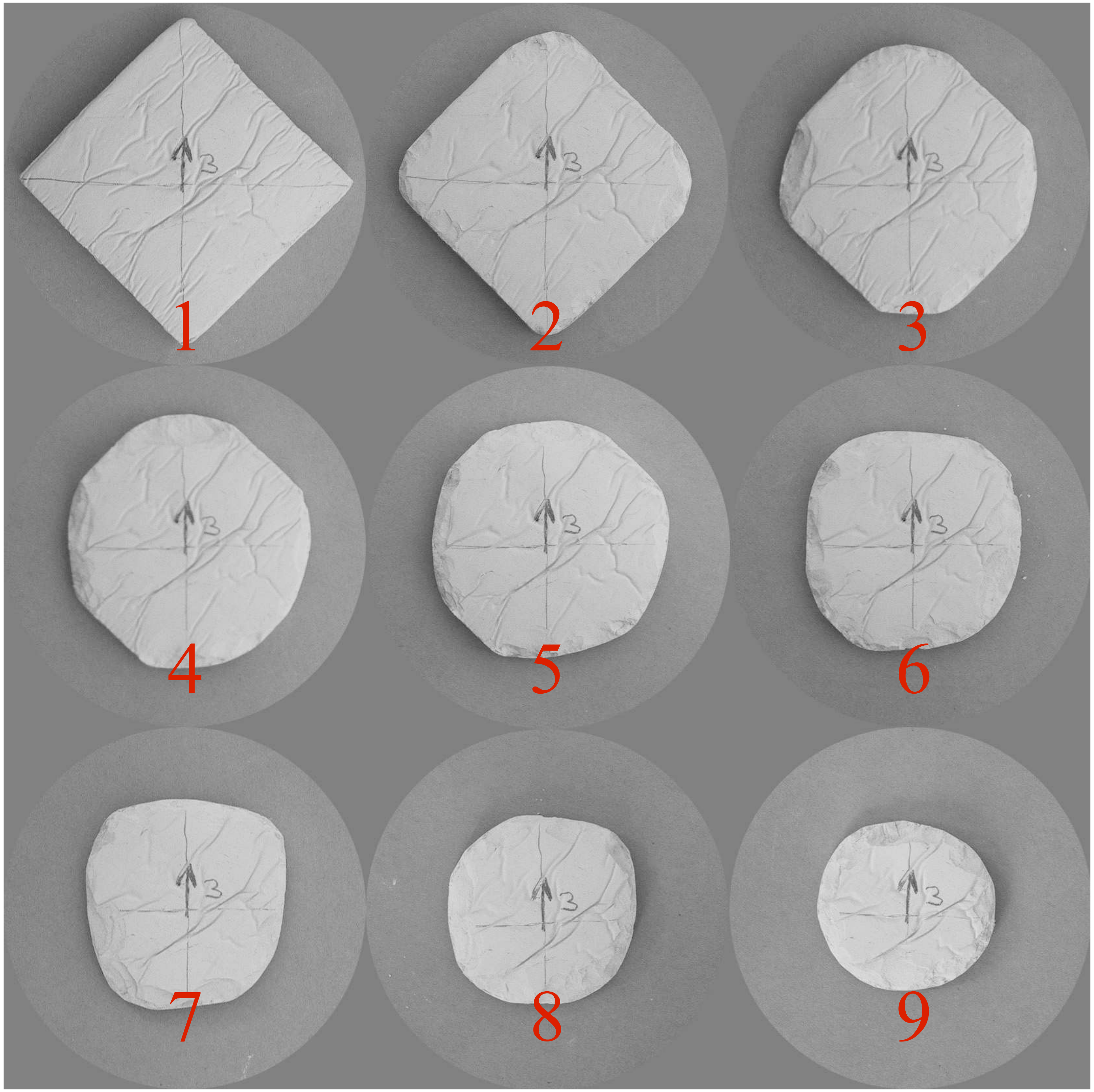
Final shape not round as found by Durian et al.

Doug Durian's Erosion Machine



Evolution of a Square Rock

Durian et al., PRL 97, 028001 (2006);
PRE 75, 021301 (2007)



What should we expect?

If $v_{\text{interface}} \propto$ local curvature,

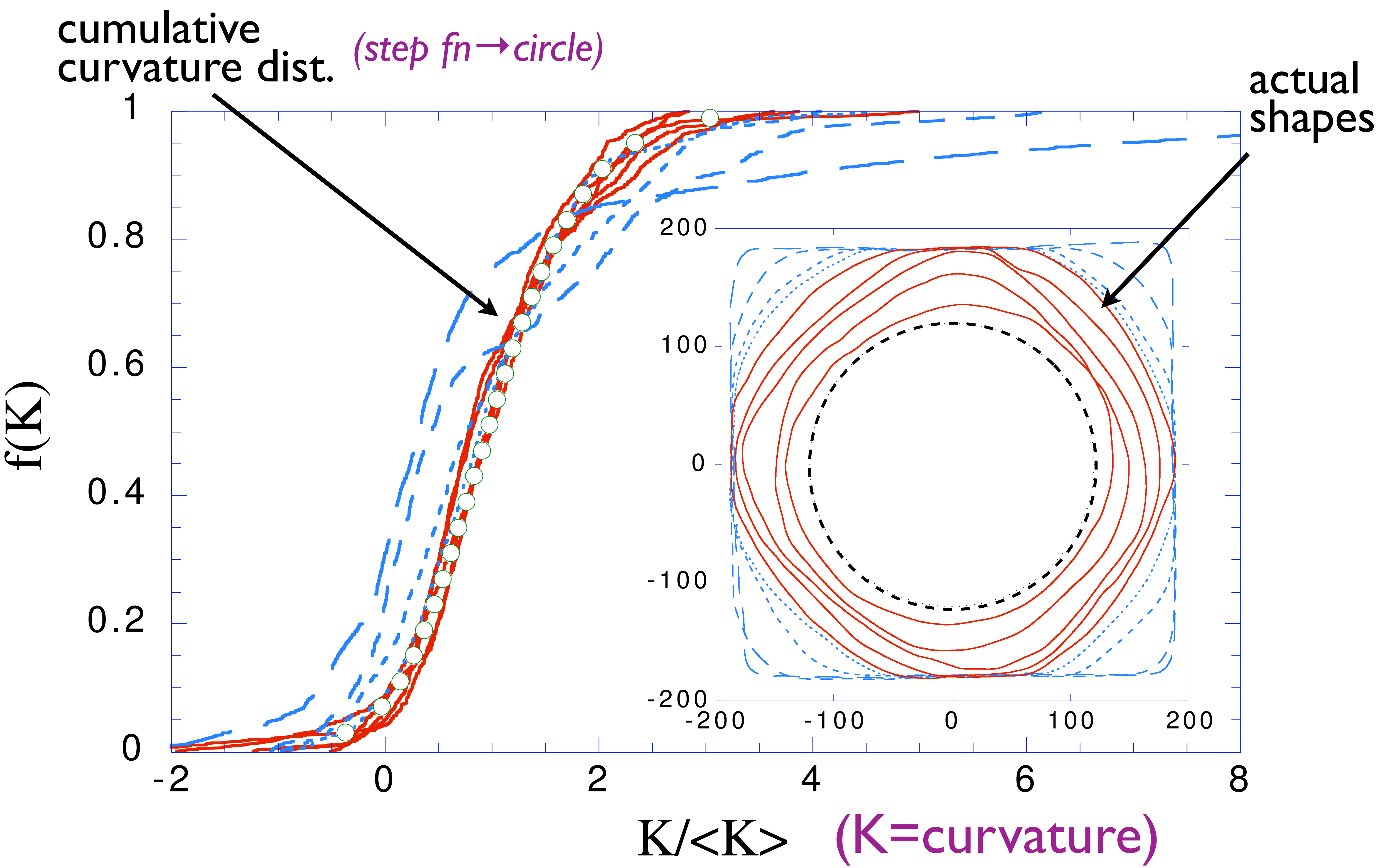
$\rightarrow$ circular final shape for $d = 2$

(not true for $d > 2$).

Mullins (1956);
many differential geometry publications

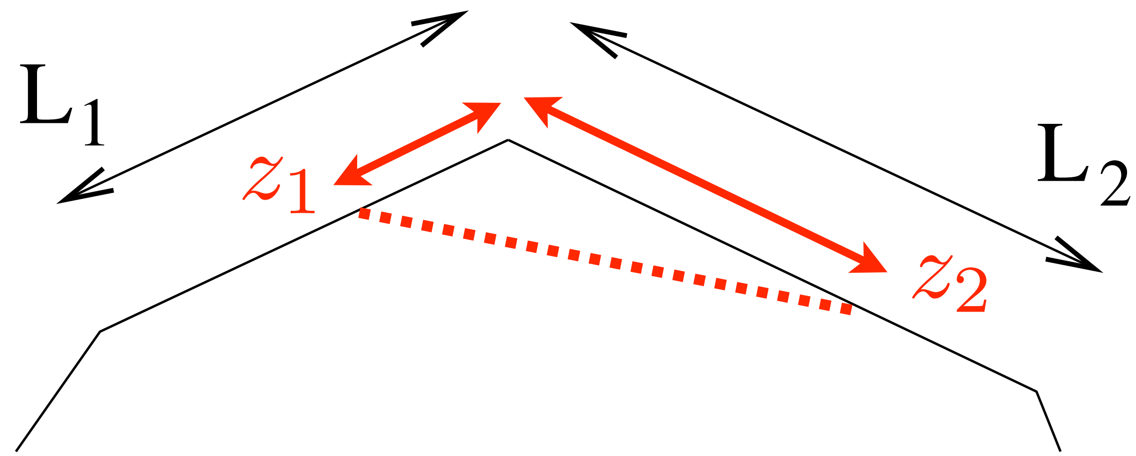
But... Final Shape is ***not*** Circular

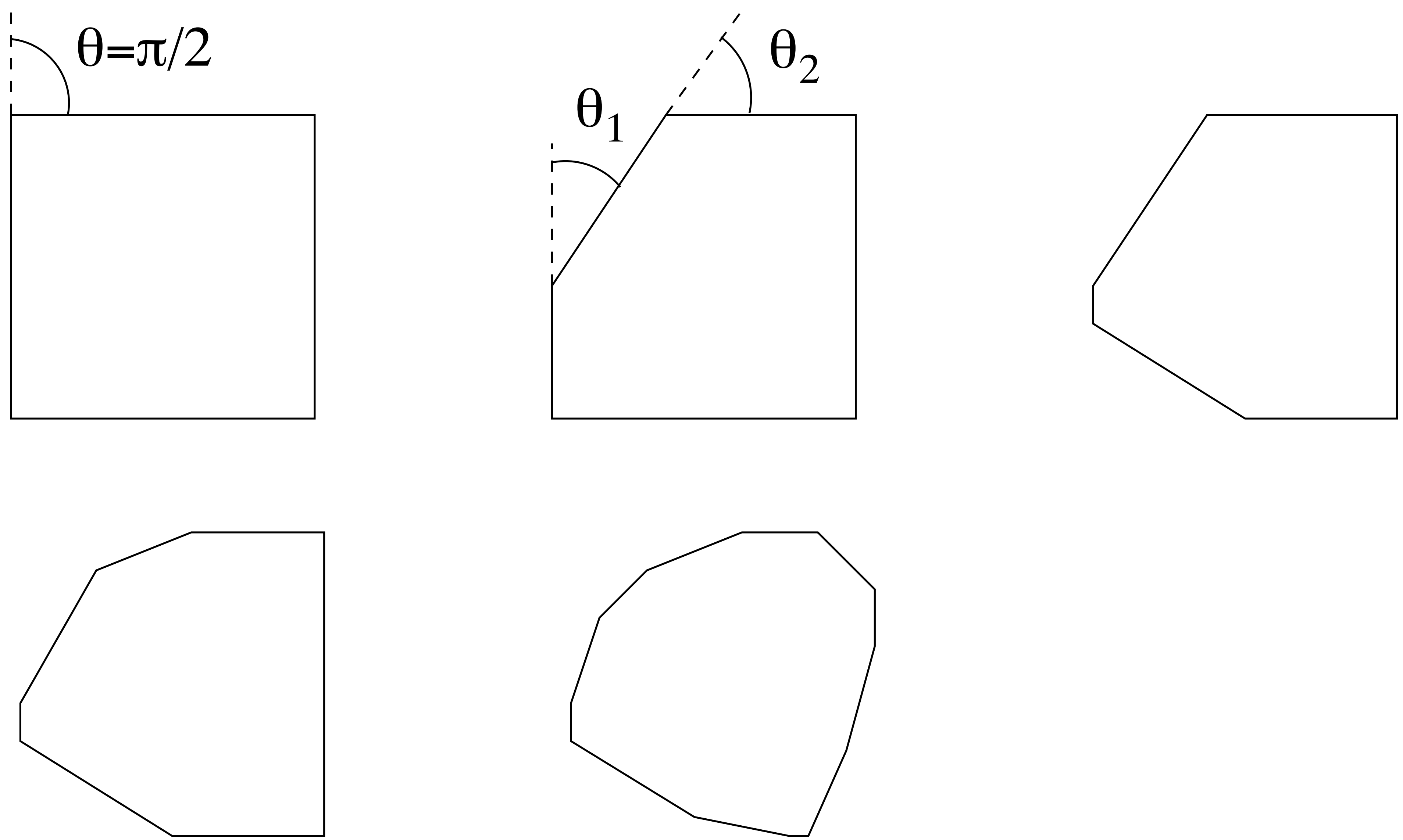
Durian et al., Phys. Rev. Lett. 97, 028001 (2006);
Phys. Rev. E 75, 021301 (2007)



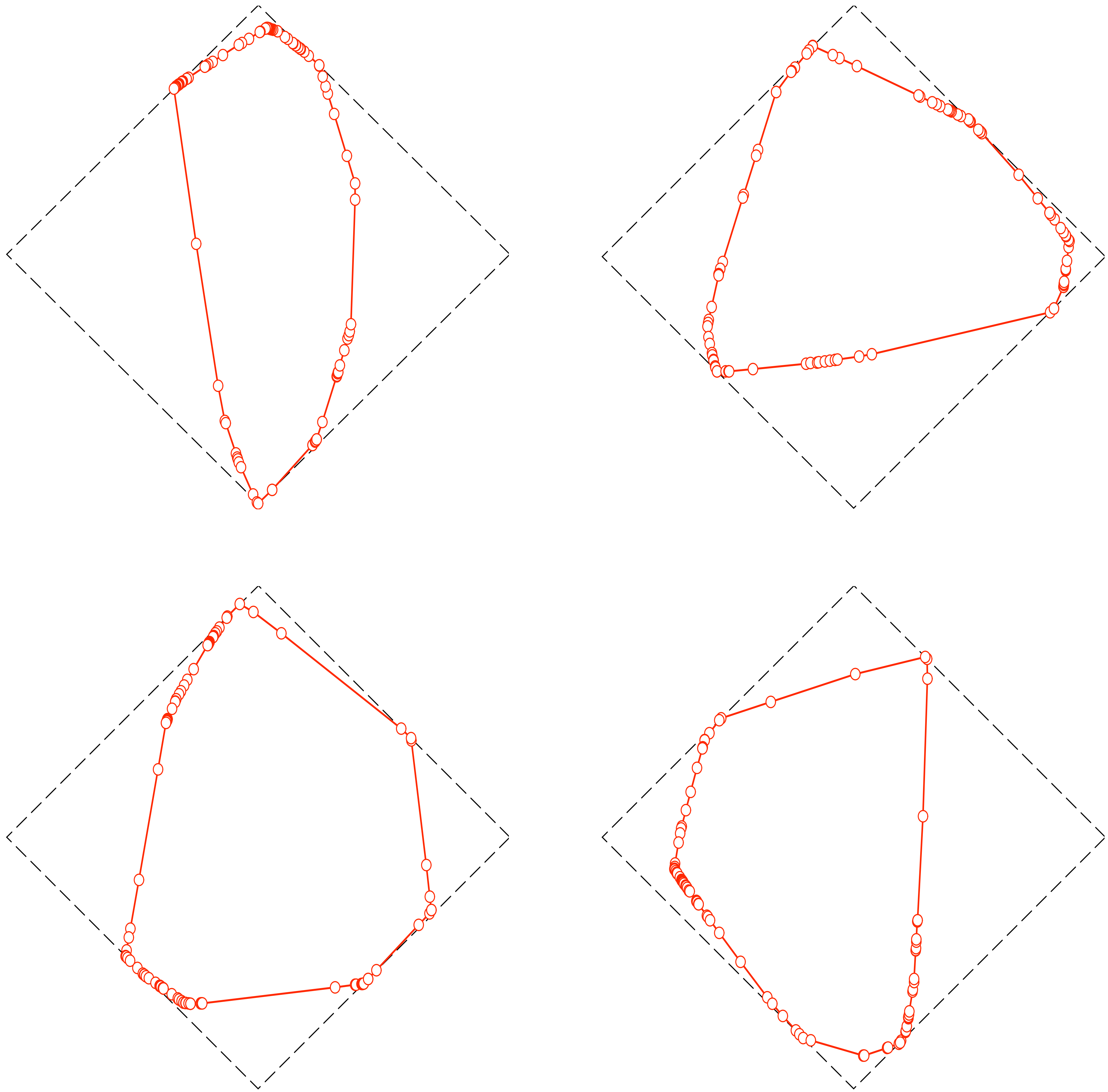
Chipping Model

geometry of single event





Numerical Realizations (100 corners)



Angle Evolution for Bisection

$n_k \equiv \#$ corners with "angle" k

$$\begin{aligned} k &\equiv -\ln_2(2\theta/\pi) \\ &= \text{number of halvings} \end{aligned}$$

Master equation: *(start with square; t+4 corners at time t)*

lose a k-corner → first term; bisect a $(k-1)$-corner → second term

$$n_k(t+1) - n_k(t) = -\frac{1}{t+4}\, n_k(t) + \frac{2}{t+4}\, n_{k-1}(t)$$

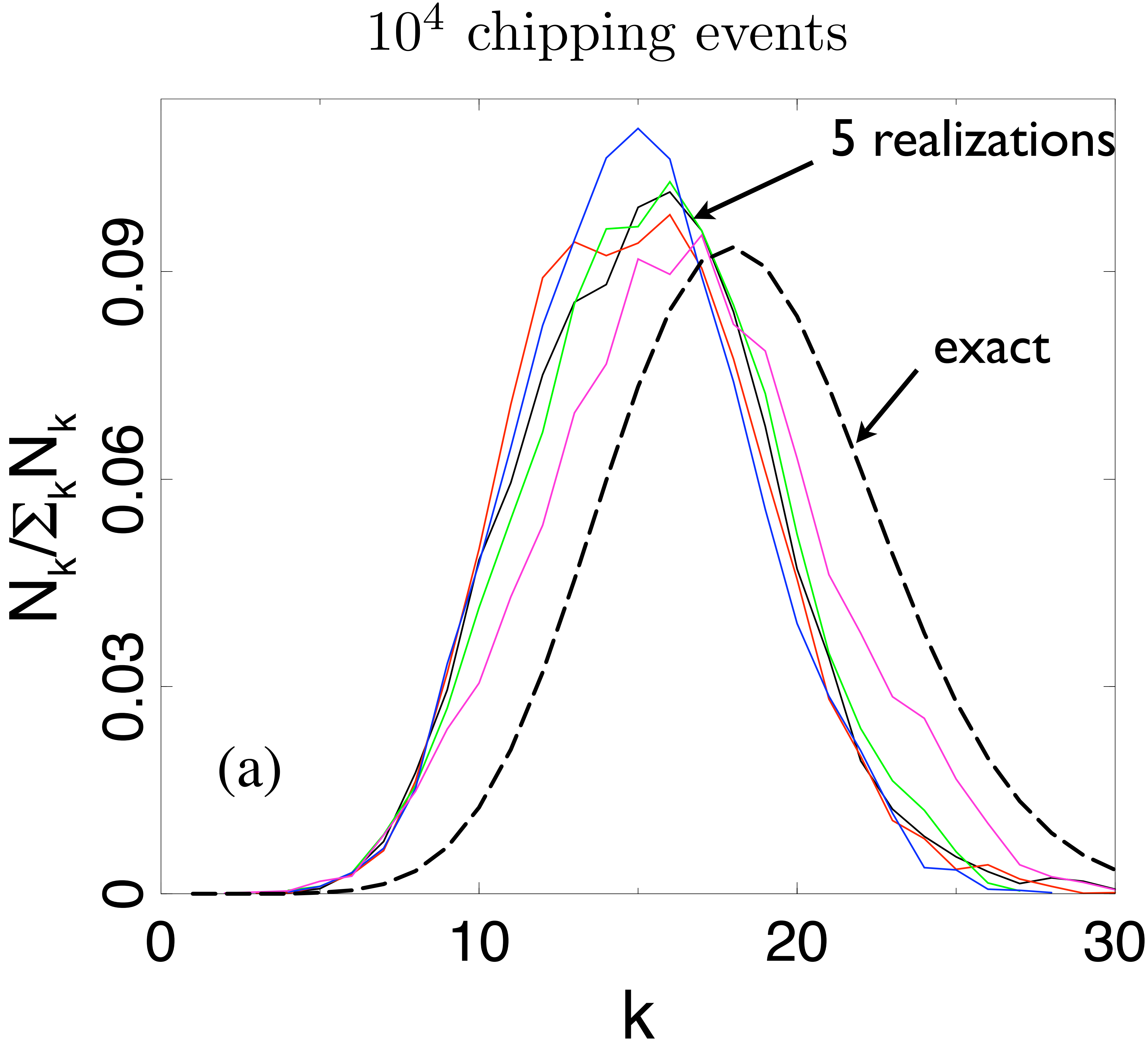
Continuum limit:

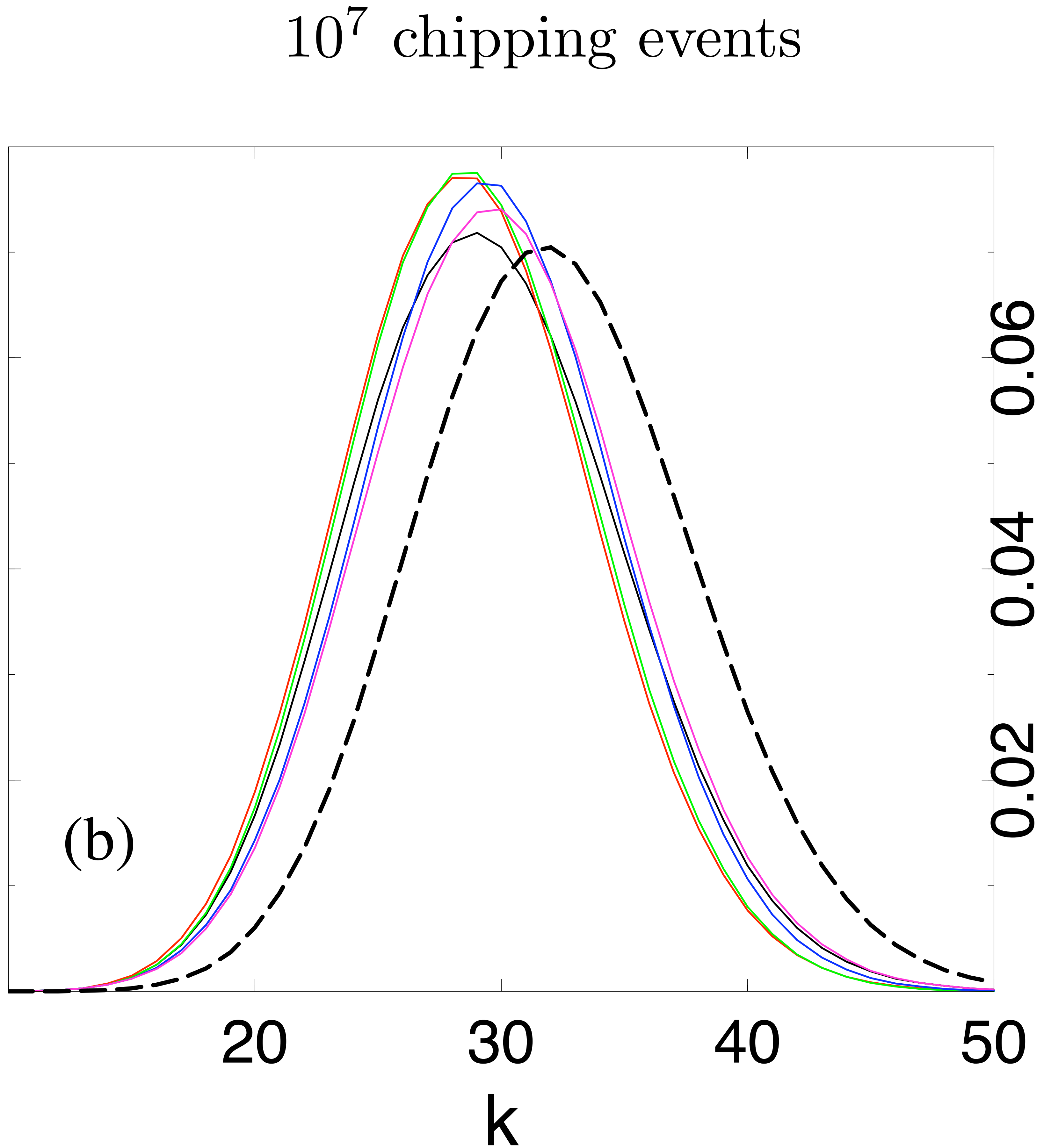
$$\frac{dn_k}{dt} = -\frac{n_k}{t} + \frac{2}{t}\, n_{k-1}$$

Result:

$$n_k(t) = \frac{12}{t}\,\frac{(2\ln t)^k}{k!}$$

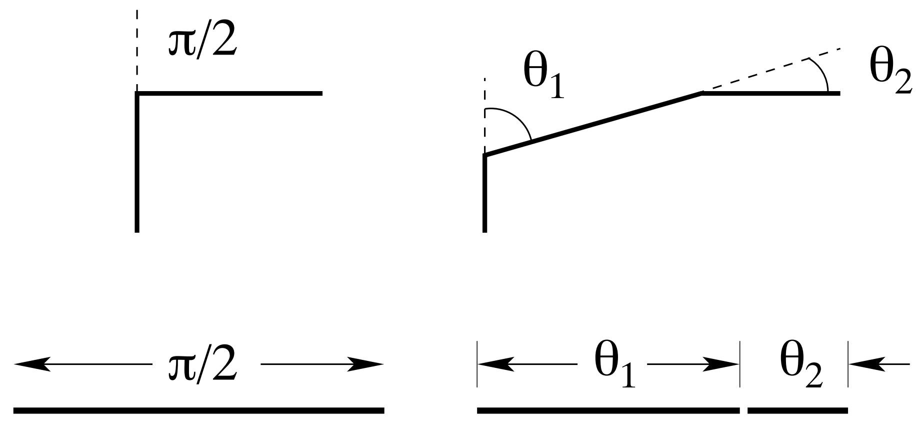
Angle Distribution for Bisection





10^7 chipping events
0.06
0.04
0.02
(b)
20
30
40
50
k

Angle Evolution for General Angles



correspondence with fragmenting a segment

$$c(x,t) = \text{fraction of angles } x = \theta/2\pi$$

$$\frac{\partial c(x,t)}{\partial t} = -c(x,t) + 2\int_x^1 c(y,t)\,\frac{dy}{y}$$

$$\frac{dn_k}{dt} = -\frac{n_k}{t} + \frac{2}{t}\,n_{k-1}$$

$$c(\theta,t) \;=\; \frac{8}{\pi}\sqrt{\frac{2t}{\ln(\pi/2\theta)}}\,e^{-t}I_1\left(\sqrt{8t\ln(\pi/2\theta)}\right) + \frac{8}{\pi}e^{-t}\delta\Big(\theta - \frac{\pi}{2}\Big),$$

$$\sim \quad e^{\sqrt{-t\ln\theta}}$$

Ziff & McGrady (1985); Ziff (1992)

broad distribution of angles

Asymmetry

$$X^2(N) = \frac{1}{N}\sum_{i=1}^{N} x_i^2 \qquad Y^2(N) = \frac{1}{N}\sum_{i=1}^{N} y_i^2$$

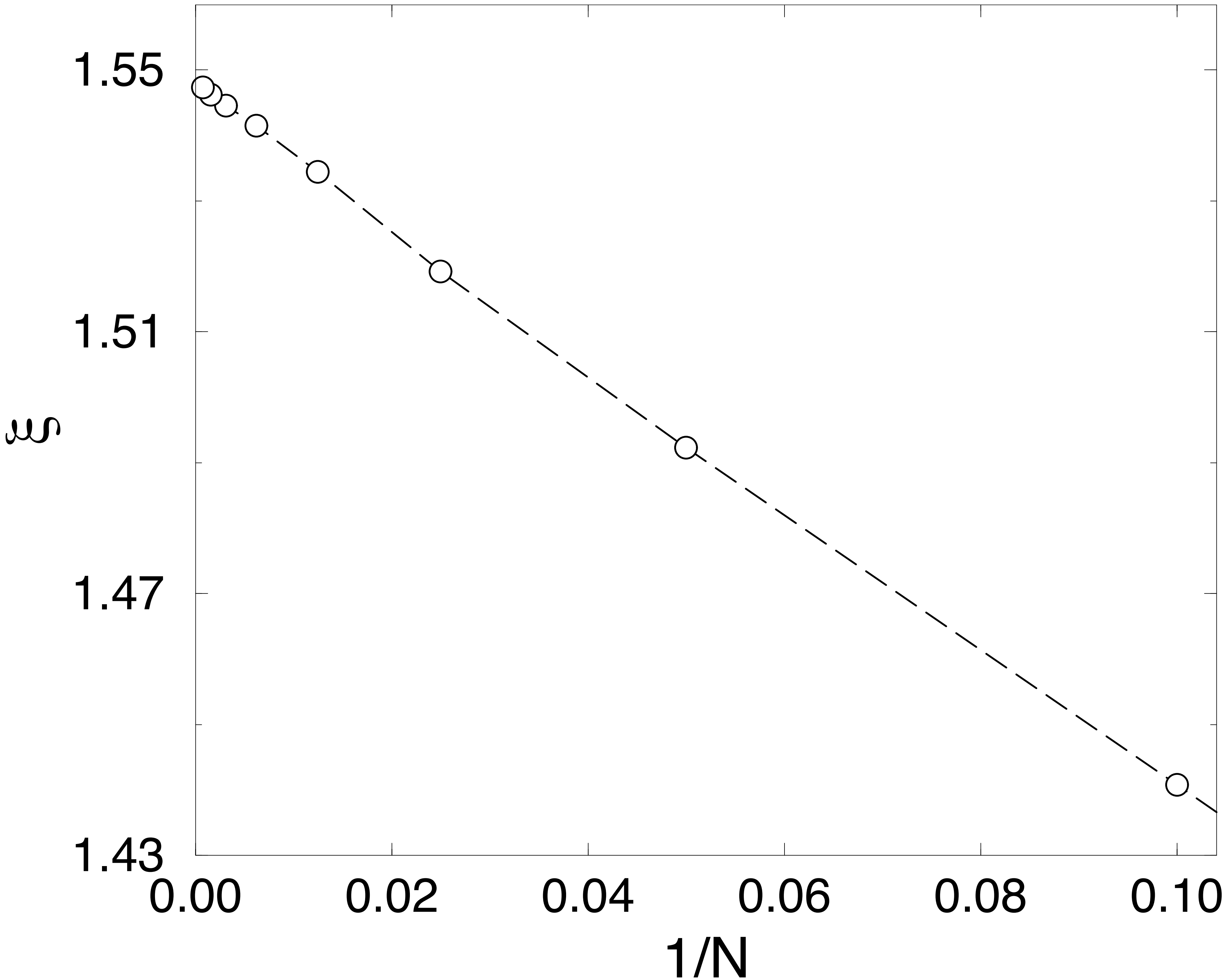
$$R_+^2(N) = \max(X^2(N), Y^2(N))$$
$$R_-^2(N) = \min(X^2(N), Y^2(N))$$

for each realization

$$\xi(N) \equiv \sqrt{\langle R_+^2(N)\rangle}/\sqrt{\langle R_-^2(N)\rangle}$$

average over all realizations

Simulation Results



Summary

Eroding rocks are ***not*** round (in d=2)

Large fluctuations between realizations

Robust with respect to extensions

preferentially chip more prominent corners

chip away more than one corner