$$\frac{\partial P}{\partial t} + v\frac{\partial P}{\partial x} = D\frac{\partial^2 P}{\partial x^2}, \quad P(x,t=0) = \delta(x)$$

$$\int_{-\infty}^{\infty}\left(\frac{\partial P}{\partial t} + v\frac{\partial P}{\partial x} = D\frac{\partial^2 P}{\partial x^2}\right)e^{ikx}\,dx$$

$$\frac{dP(k,t)}{dt} + -ikv\,P(k,t) = -k^2 D\;P(k,t)$$

$$\frac{dP(k,t)}{dt} = (ikv - k^2 D)\,P(k,t)$$

$$\Longrightarrow P(k,t) = P(k,0)\,e^{(ikv-k^2D)t}$$

$$P(k,0) = \int \delta(x)\,e^{ikx} = 1$$

$$\Longrightarrow \underline{P(k,t) = e^{(ikv-k^2D)t}}$$

$$P(x,t) = \frac{1}{2\pi}\int P(k,t)\,e^{-ikx}\,dk$$

$$= \frac{1}{2\pi}\int e^{(ikv-k^2D)t-ikx}\,dk$$

$$= \underline{\frac{1}{\sqrt{4\pi Dt}}\,e^{-\frac{(vt-x)^2}{4Dt}}}$$

$$s\,P(x,s) - \delta(x) = -v\frac{dP(x,s)}{dx} + D\frac{d^2P(x,s)}{dx^2}$$

$$P(x,s) = A e^{ax} + B e^{bx}$$

$$a = \frac{v + \sqrt{v^2+4sD}}{2D}$$

$$b = \frac{v - \sqrt{v^2+4sD}}{2D}$$

$$x>0, \quad P(x,s) = Be^{bx}$$

$$x<0, \quad P(x,s) = Ae^{ax}$$

Use continuity to get constants

$$P(x_<) = P(x_>)$$

$$\int \left[s\,P(x,s) - \delta(x) = -v\frac{d}{dx}P(x,s) + D\frac{d^2P(x,s)}{dx^2} \right] dx$$

$$-1 = D\left(\left.\frac{dP}{dx}\right|_{x=\varepsilon} - \left.\frac{dP}{dx}\right|_{x=-\varepsilon} \right)$$

$$-1 = -2AD\,\frac{\sqrt{v^2+4sD}}{2D}$$

$$A = \frac{1}{\sqrt{v^2+4sD}}$$

$$\text{so} \quad P(x,s) = \frac{1}{\sqrt{v^2+4sD}}\, e^{-\frac{v \pm \sqrt{v^2+4sD}}{2D}|x|}$$