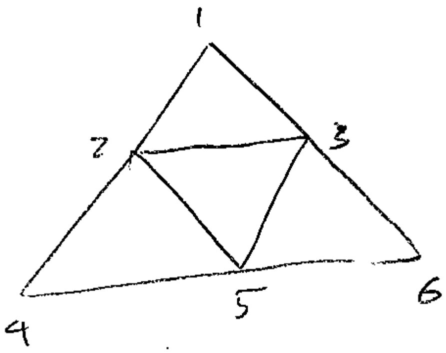


$$sP_1 = P_2 + P_3 - 2P_1 + 1$$
$$sP_4 = P_2 + P_5 - 2P_4$$
$$sP_6 = P_3 + P_5 - 2P_6$$

$$sP_2 = P_3 + P_5 + P_1 + P_4 - 4P_2$$
$$sP_3 = P_2 + P_5 + P_1 + P_6 - 4P_3$$
$$sP_5 = P_2 + P_3 + P_4 + P_6 - 4P_5$$

replace P_1, P_4, + P_6.

$$P_1 = \frac{P_2 + P_3 + 1}{(s+2)} \qquad P_4 = \frac{P_2 + P_5}{(s+2)} \qquad P_6 = \frac{P_3 + P_5}{(s+2)}$$

$$(s+4)P_2 = P_3 + P_5 + \frac{1}{(s+2)}\left(2P_2 + P_3 + P_5 + 1\right)$$
$$(s+4)P_3 = P_2 + P_5 + \frac{1}{(s+2)}\left(P_2 + 2P_3 + P_5\right)$$
$$(s+4)P_5 = P_2 + P_3 + \frac{1}{(s+2)}\left(P_2 + P_3 + 2P_5\right)$$

$$\begin{array}{ccc|c} P_2 & P_3 & P_5 & \text{const} \\ \hline s+4-\frac{2}{(s+2)} & -1-\frac{1}{(s+2)} & -1-\frac{1}{(s+2)} & \frac{1}{s+2} \\ -1-\frac{1}{s+2} & s+4-\frac{2}{(s+2)} & -1-\frac{1}{(s+2)} & \frac{1}{s+2} \\ -1-\frac{1}{s+2} & -1-\frac{1}{s+2} & s+4-\frac{2}{(s+2)} & 0 \end{array}$$

rewrite $\quad A = 1 + \frac{1}{s+2} \qquad B = s+4-\frac{2}{s+2} \qquad C = \frac{1}{s+2}$

$$\begin{array}{cccc} & P_2 & P_3 & P_5 \\ \mathrm{I} & B & -A & -A \\ \mathrm{II} & -A & B & -A \\ \mathrm{III} & -A & -A & B \end{array} \qquad \begin{bmatrix} C \\ C \\ 0 \end{bmatrix}$$

$\mathrm{II} - \mathrm{I} \Rightarrow P_2 = P_3$

$\mathrm{III}: \quad 2AP_{2/3} = BP_5 \rightarrow \quad P_5 = \frac{2A}{B} P_{2/3}$

$\mathrm{II}: \quad P_{2/3}(B-A) - \frac{2A^2}{B} P_{23} = C \quad \Rightarrow \quad P_{23} = \frac{C}{B - A - \frac{2A^2}{B}}$

$$P_{23} = \frac{CB}{B^2 - BA - 2A^2} = \frac{CB}{(B-2A)(B+A)}$$

$aB + aA + bB - b2A = 1$

$a+b=1 \qquad a - \cancel{2b} = \cancel{0}\ 2b$

$a = \frac{1}{3} \quad b = \frac{2}{3}$

$$P_{23} = \frac{C}{3(B-2A)} + \frac{2C}{3(B+A)}$$

$B-2A = s+4-\frac{2}{s+2} - 2 - \frac{2}{s+2} = s+2-\frac{4}{s+2} \qquad B+A = s+5-\frac{1}{s+2}$

$$P_{23} = \frac{2\,\frac{1}{(s+2)}}{3\left[(s+2) - \frac{4}{(s+2)}\right]} + \frac{\frac{1}{(s+2)}}{3\left[(s+5) - \frac{1}{(s+2)}\right]}$$

$$= \frac{2}{3\left[(s+2)^2 - 4\right]} + \frac{1}{3\left[(s+5)(s+2) - 1\right]}$$

$a^2 - b^2 = (a+b)(a-b)$

$$= \frac{2}{3\left[s(s+4)\right]} + \cancel{\frac{1}{3\left[s^2 + 7s + 9\right]}} + \cancel{\frac{1}{(s+3.5)^2 - 3.25}} + \frac{1}{\left(s+3.5+\sqrt{3.25}\right)\left(s+3.5-\sqrt{3.25}\right)}$$

$A = 3.5 + \sqrt{3.25}$ $\quad B = 3.5 - \sqrt{3.25}$

$A - B = 2\sqrt{3.25}$

$$= \frac{2 \cdot 1}{3\left[s(s+4)\right]} + \frac{1}{(s+A)(s+B)}$$

$aS + a4 + bS = 1$

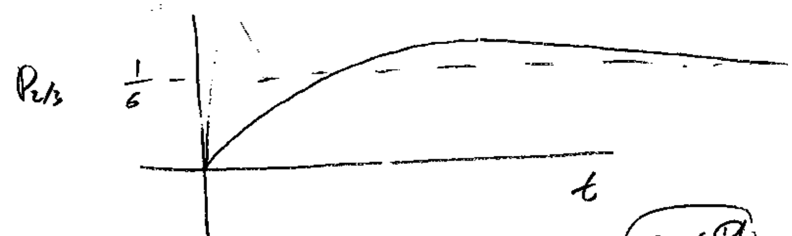
$a = \frac{1}{4} \quad b = -\frac{1}{4}$

$aS + aB + bS + bA = 1$

$b = -a \quad a = -\frac{1}{A-B}$

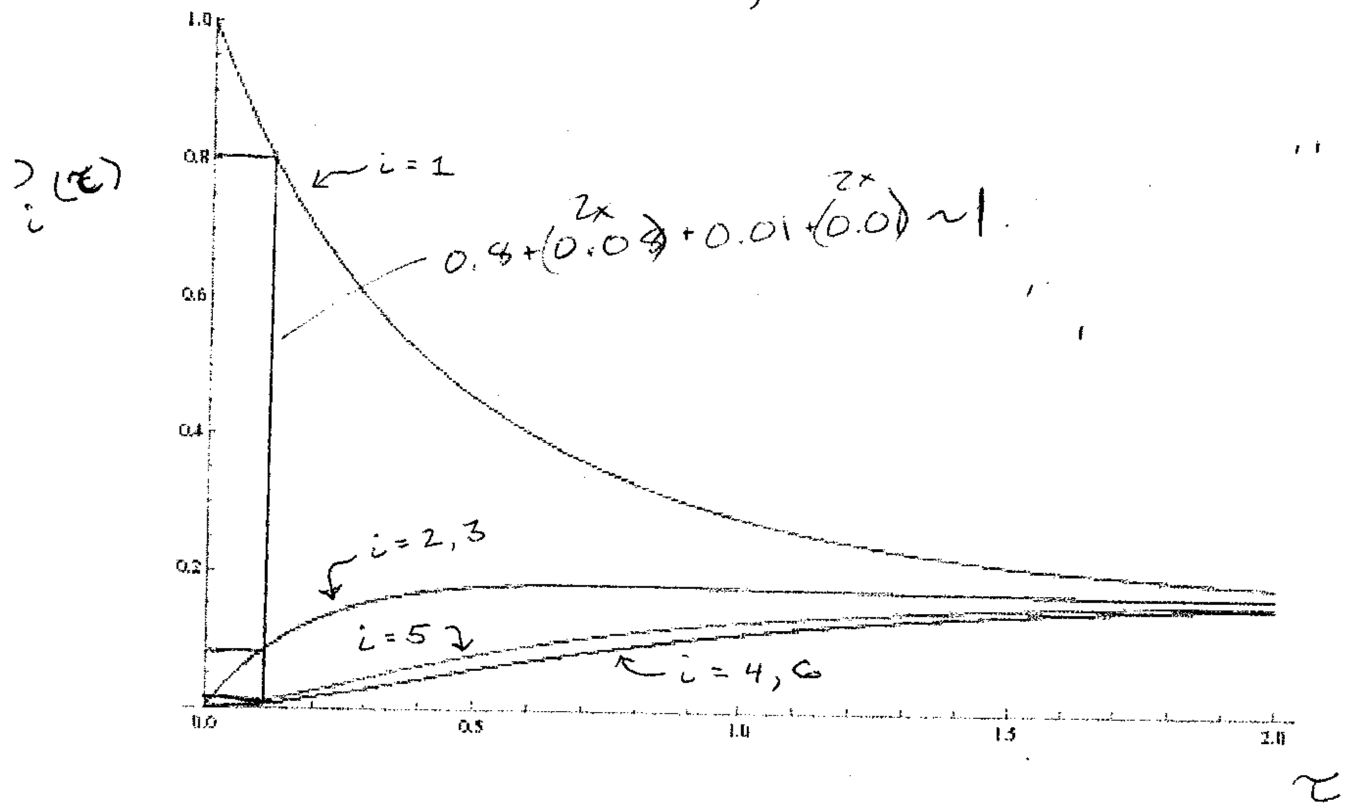
$$= \frac{1}{6}\left[\frac{1}{s} - \frac{1}{s+4}\right] + \frac{1}{(A-B)}\left[\frac{1}{S+B} - \frac{1}{S+A}\right]$$

$$P_{2/3}(t) = \frac{1}{6}\left(1 - e^{-4t}\right) + \frac{1}{2\sqrt{3.25}}\left(e^{-\left(3.5-\sqrt{3.25}\right)t} - e^{-\left(3.5+\sqrt{3.25}\right)t}\right)$$



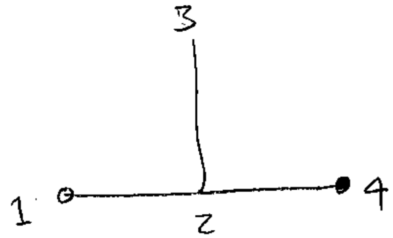
continue logic to get other tie dependancies P_1, P_5, $P_{4/6}$

First Order Sierpinski Gasket



$\tau \equiv \omega t$

(2a)



① $sP_1 = P_2 - P_1 + 1$

② $sP_2 = P_1 + P_3 - 3P_2$

③ $sP_3 = P_2 - P_3$

④ $sP_4 = P_2$

solve for P_2:

① $P_1 = \dfrac{P_2 + 1}{s+1}$ ④ $P_4 = \dfrac{P_2}{s}$ ③ $P_3 = \dfrac{P_2}{(s+1)}$

plug into ②: $P_2 = \dfrac{1}{s(s+1) + 3(s+1) - 2} = \dfrac{1}{(s+1)(s+3) - 2}$

(2a)

$$P_2 = \frac{1}{s(s+1)+3(s+1)-2} = \frac{1}{s^2+s+3s+3-2} = \frac{1}{s^2+4s+1} = \frac{1}{(s+2)^2-3}$$

using $a^2-b^2 = (a+b)(a-b)$

$$P_2 = \frac{1}{(s+2+\sqrt{3})(s+2-\sqrt{3})} \quad \text{split} \quad \frac{a}{(s+2+\sqrt{3})} + \frac{b}{(s+2-\sqrt{3})}$$

$$aS + a(2-\sqrt{3}) + bS + b(2+\sqrt{3})$$

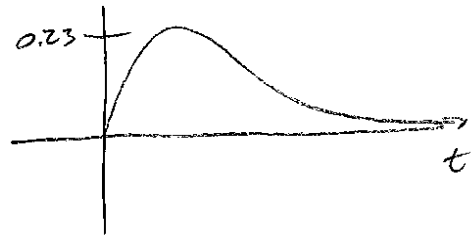
$$b = -a$$

$$a\left[2-\sqrt{3}-2-\sqrt{3}\right] = 1$$

$$a = -\frac{1}{2\sqrt{3}}$$

$$P_2 \quad -\frac{1}{2\sqrt{3}(s+(2+\sqrt{3}))} + \frac{1}{2\sqrt{3}(s+(2-\sqrt{3}))}$$

$$P_2(t) = \frac{1}{2\sqrt{3}}\left[e^{-(2-\sqrt{3})t} - e^{(2+\sqrt{3})t}\right]$$



$$P_1 = \frac{P_2}{(s+1)} + \frac{1}{(s+1)} = \frac{-1}{2\sqrt{3}(s+(2+\sqrt{3}))(s+1)} + \frac{1}{2\sqrt{3}(s+(2-\sqrt{3}))(s+1)} + \frac{1}{s+1}$$

same fraction techique.

$$aS + a + bS + b(2+\sqrt{3})$$

$$b=-a \quad a\left[1-2-\sqrt{3}\right]=1 \quad a = \frac{-1}{1+\sqrt{3}} \quad b = \frac{1}{1+\sqrt{3}}$$

$$aS + a + bS + b(2-\sqrt{3})$$

$$a = \frac{-1}{1-\sqrt{3}} \quad b = \frac{1}{1-\sqrt{3}}$$

$$P_1 = \frac{1}{(2\sqrt{3})(1+\sqrt{3})(s+(2+\sqrt{3}))} - \frac{1}{(2\sqrt{3})(1+\sqrt{3})(s+1)} - \frac{1}{2\sqrt{3}(1-\sqrt{3})(s+(2-\sqrt{3}))} + \frac{1}{2\sqrt{3}(1-\sqrt{3})(s+1)} + \frac{1}{s+1}$$

$$\boxed{P_1 = \frac{1}{2\sqrt{3}}\left[\frac{1}{1+\sqrt{3}}\left(e^{-(2+\sqrt{3})t} - e^{-t}\right) - \frac{1}{1-\sqrt{3}}\left(e^{-(2-\sqrt{3})t} - e^{-t}\right)\right] + e^{-t}}$$

2a)

$$P_4 = \frac{P_2}{s} = \frac{-1}{2\sqrt{3}} \frac{1}{(s+(2+\sqrt{3}))s} + \frac{1}{2\sqrt{3}} \frac{1}{(s+(2-\sqrt{3}))s}$$

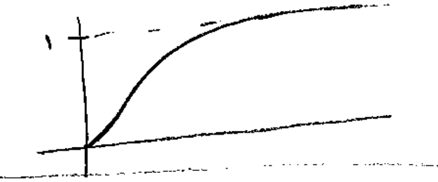
fraction technique.

$$aS + bS + b(2+\sqrt{3}) \qquad aS + bS + b(2-\sqrt{3})$$

$$b = -a \quad b = \frac{1}{2+\sqrt{3}} \qquad b = -a \quad b = \frac{1}{2-\sqrt{3}}$$

$$P_4 = \frac{1}{2\sqrt{3}(2+\sqrt{3})(s+(2+\sqrt{3}))} - \frac{1}{2\sqrt{3}(2+\sqrt{3})}\frac{1}{s} - \frac{1}{2\sqrt{3}(2-\sqrt{3})(s+(2-\sqrt{3}))} + \frac{1}{2\sqrt{3}(2-\sqrt{3})}\frac{1}{s}$$

$$P_4(t) = \frac{1}{2\sqrt{3}}\left[\frac{1}{2+\sqrt{3}}\left(e^{-(2+\sqrt{3})t} - 1\right) - \frac{1}{(2-\sqrt{3})}\left(e^{-(2-\sqrt{3})t} - 1\right)\right]$$



2b)

$$\langle t \rangle = \int_0^\infty t\, F_4$$ where F_4 is the first travel probability.

$$F_4 = P_2$$

$$\langle t \rangle = \int_0^\infty t\, P_2$$

use the generating function $\left.\frac{\partial^n}{\partial^n(-s)} P_2(s)\right|_{s=0} = \left.\frac{\partial^n}{\partial^n(-s)} \int_0^\infty P_2(t)e^{-st}\,dt\right|_{s=0} = \int_0^\infty t^n P_2(t)\,dt$

$$\left.\frac{\partial}{\partial(-s)} \frac{1}{(s+1)(s+3)-2}\right|_{s=0} = \left.\frac{-1}{((s+1)(s+3)-2)^2}\left[(s+1)+(s+3)\right]\right|_{s=0} = \boxed{4}$$

(2c) assume $2 \to 3 = r \neq w$.

$$① \; sP_1 = P_2 - P_1 + 1$$
$$② \; sP_2 = P_1 + P_3 - 2P_2 - rP_2$$
$$③ \; sP_3 = rP_2 - P_3$$
$$④ \; sP_4 = P_2$$

$$① \; P_1 = \frac{P_2+1}{s+1} \qquad ③ \; P_3 = \frac{rP_2}{(s+1)} \qquad ④ \; \frac{P_2}{s}$$

plug into ②

$$sP_2 = \frac{P_2+1}{(s+1)} + \frac{rP_2}{(s+1)} - P_2(2+r)$$

$$s(s+1)P_2 = P_2 + 1 + rP_2 - P_2(2+r)(s+1)$$

$$P_2 = s(s+1) - (1+r) + (2+r)(s+1)$$

$$P_2 = \frac{1}{s(s+1) + \underbrace{(2+r)}(s+1) - \underbrace{(1+r)}} = \frac{1}{s^2 + s(3+r) + 1}$$

This changes the equations such that $2+\sqrt{3} \to A+\sqrt{B}$ and $2-\sqrt{3} \to A-\sqrt{B}$ $\quad 2 \to A$, $\sqrt{3} \to B$

when $A = \frac{3+r}{2}$ $\quad B = \frac{(3+r)^2}{4} - 1$

Part (b) would be

$$\frac{\partial}{\partial(-s)} \frac{1}{s^2 + s(3+r)+1}\Bigg|_{s=0} = \frac{1}{(s^2+s(3+r)+1)^2}\left[2s+3+r\right]\Bigg|_{s=0} = \boxed{3+r}$$

$r \ll w$ then $t = 3$
$r \gg w$ then $t = r$

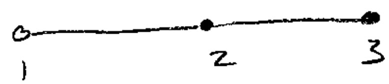
(2c)

Therefore

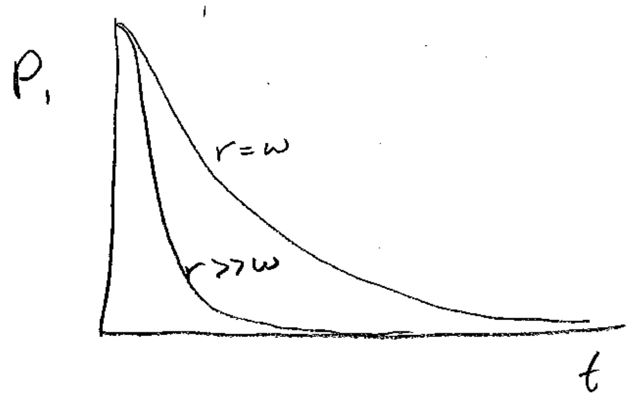
$$P_1 = \frac{1}{A\sqrt{B}}\left[\frac{1}{A-1+\sqrt{B}}\left(e^{-(A+\sqrt{B})t} - e^{-t}\right) - \frac{1}{A-1-\sqrt{B}}\left(e^{-(A-\sqrt{B})t} - e^{-t}\right)\right] + e^{-t}$$

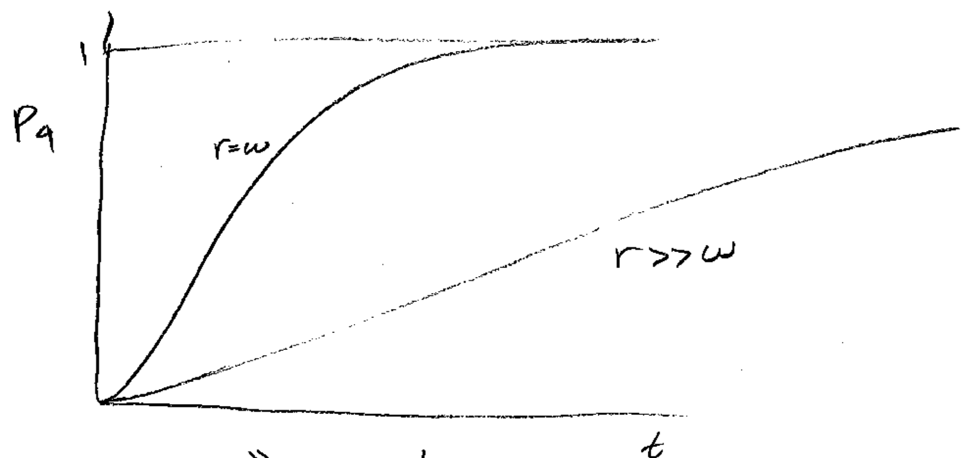
$$P_4 = \frac{1}{A\sqrt{B}}\left[\frac{1}{A+\sqrt{B}}\left(e^{-(A+\sqrt{B})t} - 1\right) - \frac{1}{A-\sqrt{B}}\left(e^{-(A-\sqrt{B})t} - 1\right)\right]$$

$r \ll \omega$ set $r = 0$ $A = \frac{3}{2}$ $B = \frac{5}{4}$ this leads to the problem with no node 3, or



$r \gg \omega$ then P_1 drops to 0 much quicker and P_4 grows to 1 much slower than if $r = \omega$





this is because the particle gets "stuck" at position 3