

Assignment #5 PY 542 Week of September 20–October 3, 2008

Reading: After some concluding discussion on kinetic spin systems, our next topic will be the Langevin equation. Excellent treatments are given in books of Reif and van Kampen, now both on reserve.

Notes: There will be no office hours and no class on Tuesday September 30 (Jewish New Year, 5769). To compensate there will be lectures on Wednesday 2–3:30 and the regular Thursday lecture time.

Problems: Friday October 10 by 4pm in Andrew Inglis' mailbox.

1. Consider the voter model on an infinite line in which voter i^{th} consults its right neighbor ($i + 1$) and second neighbor ($i + 2$). Write the equation of motion for the mean spin and solve it for the case where the spin at the origin is initially in the $+1$ state and the states of all other spins have zero average value.
2. For the Langevin equation of a Brownian particle of mass m , compute the mean-square displacement $\langle [x(t) - x(0)]^2 \rangle$ in terms of kT , m , the damping coefficient γ , and the initial speed v_0 .
3. A rod-like molecule rotates freely in a plane, and is subject to a Langevin force $L(t)$ due to its surroundings. The equation of motion for the orientation angle ϕ of the molecule is $\ddot{\phi} + \gamma\dot{\phi} = L(t)$. Determine the correlation function for the x -component of the dipole moment of the molecule $\langle \cos \phi(t_1) \cos \phi(t_2) \rangle$.
4. Consider the Edwards-Wilkinson (EW) model for the growth of a $(d - 1)$ -dimensional surface by sequential, random particle deposition. The Langevin equation for the deviation of the height from its mean value, $h(\vec{x}, t)$, is

$$\frac{\partial h(\vec{x}, t)}{\partial t} = \nabla^2 h(\vec{x}, t) + \eta(\vec{x}, t)$$

The first term on the right is a surface tension that smooths the surface. The second (noise) term accounts for fluctuations due to particle deposition, with $\langle \eta(\vec{x}, t) \rangle = 0$, and $\langle \eta(\vec{x}, t) \eta(\vec{x}', t') \rangle = \delta(\vec{x} - \vec{x}') \delta(t - t')$. Compute the height-height correlation function $\langle h(\vec{x}, t) h(\vec{x}', t) \rangle$. Determine whether the surface is smooth or rough in the long-time limit as a function of d .

Note: The solution to the EW model is readily available in the literature. Please, no peeking! Let your solution represent your personal effort. As a hint, it will be helpful to Fourier transform the equation of motion.