Assignment #4 PY 542 Week of September 22 – 26, 2008

Reading: This week, we will begin discussing the evolution of spin systems. Please read chapter 7 of my book "Fundamental Kinetic Processes" posted on the course website. I've also posted Glauber's original article on the course website. I would be very grateful to receive your comments for improvements, suggestions for revisions, and to know about any typos.

Note: My Tuesday office hour is cancelled because I'm hosting the colloquium speaker. If you would like to see me, please make an appointment. I also plan to give the midterm on Thursday October 23, a day when I will be out of town. Please let me know if there is any problem with that date.

Problems: Due Thursday October 2.

1. Consider a continuous-time random walk on a semi-infinite chain $n \geq 0$, with transition rate r for rightward hops and $\ell > r$ for leftward hops. At the origin, hopping to the left does not occur. Compute $P_n(t)$, the probability that site n is occupied at time t, for the initial condition $P_n(t=0) = \delta_{n,0}$. *Note:* You may find it helpful to consider the special case of the limit $t \to \infty$.

2. Consider a homogeneous continuous-time walk on an infinite one-dimensional chain with right- and left-hopping rates $r_n = r$, $\ell_n = \ell$ $\forall n$ with initial condition $P(n, t=0) = \delta_{n,0}$.
   (a) Find $P_n(t)$ for: (i) $r = \ell$ and (ii) $r \neq \ell$.
   (b) Determine the mean displacement $\langle x(n) \rangle$ and the mean-square displacement $\langle x^2(n) \rangle$ for both $r = \ell$ and (ii) $r \neq \ell$.

   Useful formulae:
$$e^{z\cos\theta} = \sum_{n=0}^{\infty} I_n(z) e^{in\theta}$$
$$e^{z\sin\theta} = I_0(z) + 2\sum_{n=0}^{\infty}(-1)^n I_{2n+1}(z)\sin[(2n+1)\theta] + 2\sum_{n=0}^{\infty}(-1)^n I_{2n}(z)\cos(2n\theta)$$

3. Consider a symmetric random walk in one dimension in which the length of the n^{th} step is λ^n.
   (a) Compute the probability distribution in the limit $n \to \infty$ for the case $\lambda = 1/2$.
   (b) More ambitiously, compute the probability distribution in the limit $N \to \infty$ for the case $\lambda = 1/\sqrt{2}$.

   Note: You may find it helpful to work out the exact probability distribution by enumeration for a small number of steps.

4. Consider the voter model on a d-dimensional hypercubic lattice. Suppose that a single voter at the origin is initially in the $+1$ state, while all other voters have no net opinion. That is, $\langle \sigma(\mathbf{x}, 0) \rangle = \delta(\mathbf{x})$. Evaluate the time evolution of the spin at the origin.