Assignment #2 PY542 Week of September 8–12, 2008

Reading: This week, we will discuss the Chapman-Enskog expansion of the Boltzmann transport equation, the calculation of transport coefficients, the connection to hydrodynamics and them some basic hydrodynamic phenomena. Chapter 5 of Huang's book gives a nice treatment of these topics. Chapters 13 & 14 of Reif are helpful for the calculation of transport coefficients. Both these book are on reserve at the Science and Engineering library.

Notes: Regular schedule this week and discussion section Wednesday from 2–3pm.

Problems: Due Thursday September 18.

1. Small spherical particles of mass m are suspended in an equilibrium fluid at temperature T. A uniform gravitational force that points in the $-z$ direction, with $z = 0$ corresponding to the ground.
   (a) Write the Boltzmann transport equation for molecules in this gas. Show that the density n as a function of height z is $n(z) = n_0 e^{-\lambda z}$. Determine λ as a function of m, g, and kT.
   (b) Suppose that the particles are influenced both by gravity and a viscous force $\vec{F} = -(6\pi\eta R)\vec{v}$, where η is the viscosity coefficient and R is the particle radius. What is the limiting speed v_∞ of the particles in the gravitational field?
   (c) Due to the density gradient, particles diffuse to decrease the gradient. If the diffusion coefficient is D, determine the diffusive current j.
   (d) Consider both the effects of gravitational settling and diffusion and show that D and η obey the Einstein relation $D = kT/(6\pi\eta R)$.

2. Assume that the Earth started at a uniform temperature T_0 and that it cooled by heat diffusion to its surface that is maintained at $T = 0°\text{C}$, and then dissipated into empty space. Approximate the Earth as a semi-infinite medium in the region to $z < 0$, with empty space for $z > 0$.
   (a) Show that the temperature in the region $z < 0$ is given by

$$T(z,t) = \frac{T_0}{\sqrt{4\pi\kappa t}} \int_{-\infty}^{0} e^{-(z-z')^2/4\kappa t} dz'.$$

   This form is a solution to the heat diffusion equation with boundary condition $T(z = 0, t) = 0$ and κ is the heat diffusion coefficient.
   (b) Compute the temperature gradient $\frac{dT}{dz}$ at the Earth's surface and estimate its value numerically. What might be wrong with your calculation, given that the correct answer is $\frac{dT}{dz} \approx -3\times 10^{-2}$°C/m, $\kappa \approx 3\times 10^6$ m^2/sec, and $T_0 \approx 4000°\text{C}$.
   (c) Write an expression for $T(r,t)$ for a spherically symmetric Earth of radius R with its surface at $T = 0$ and that its interior initially at $T = 4000°\text{C}$.