Assignment #1 PY 542 Week of September 1–5, 2008

Reading: The course begins with linear transport theory. This first topic is discussed, *e.g.*, in Reif's book, and the relevant section of this book is posted on the course website. The next course topic is the Boltzmann equation. Here, I am roughly following chapters 3 – 5 of Huang's book, and a book chapter on the subject, written by E. Cohen is available on the course website. Most reasonable texts in statistical mechanics should cover these two topics.

Problems: Due Thursday September 11.

1. (a) Derive $\langle v_x^{2n} \rangle$ for an ideal case for arbitrary positive integer n. (b) Using the same considerations as in lecture, estimate the speed of the *slowest* molecule in a gas of N molecules.

2. (*Reif, 12.12*) A long cylindrical wire of radius A and electrical resistance R per unit length is stretched along the axis of a long cylindrical container of radius $B > A$. The outer wall of the container is maintained at a fixed temperature T_0 and is also filled with a gas of thermal conductivity κ. Compute the temperature difference ΔT between the wire and the container walls when a small electrical current I passes through the wire. Show also that knowledge of ΔT provides the thermal conductivity of the gas.

3. A container is divided into two compartments. One side contains an ideal gas of mass density ρ and temperature T, while there is vacuum on the other side. A hole of radius a (much less than the mean-free path) is opened. (a) Compute exactly the flux of particles through the hole. (b) After a short time, the hole is closed. Under the assumption that that relatively few particles passed through the hole, what is the mean speed of particles that have "escaped"? Determine the temperature of the escaped particles T' in terms of T. Give a physical explanation of your result.

4. Under the (incorrect) assumption of isothermal conditions, estimate the density difference in the atmosphere between sea level and the top of Mt. Everest (approx. 8800 meters).

   Suppose that a 8800 meter horizontal column of air is prepared with the initial density profile appropriate for a vertical column. Estimate roughly how much time is needed before the density in the column is spatially constant. (You will need to construct a reasonable definition of spatially constant.)

5. Consider the formation of a layer of ice on the top of pond in the winter. Assume that the water in the pond is always at temperature 0°C, while the air above the pond is fixed at a temperature $T_0 < 0$. (a) How does the thickness of the ice grow with time? (b) Using reasonable numerical values and $T_0 = -10$°C, try to estimate numerically how the ice thickness grows with time. Make simplifying assumptions wherever needed.