7.5. (a) We have to evaluate the quantities

$$\kappa_T = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_T \quad \text{and} \quad \kappa_S = \frac{1}{n}\left(\frac{\partial n}{\partial P}\right)_z ,$$

where $n = N/V$. For $N_0 \ll N$, $n(T,z) = aT^{3/2} g_{3/2}(z)$, where a is a constant; see eqn. (7.1.8). It follows that

$$dn = \frac{3}{2} aT^{1/2} g_{3/2}(z)\, dT + aT^{3/2}\left\{\frac{1}{z} g_{1/2}(z)\right\} dz .$$

Similarly, since $P = cT^{5/2} g_{5/2}(z)$, where c is a constant,

$$dP = \frac{5}{2} cT^{3/2} g_{5/2}(z)\, dT + cT^{5/2}\left\{\frac{1}{z} g_{3/2}(z)\right\} dz .$$

The quantities κ_T and κ_S are then given by

$$\kappa_T = \frac{1}{n}\frac{a}{cT}\frac{g_{1/2}(z)}{g_{3/2}(z)} \quad \text{and} \quad \kappa_S = \frac{1}{n}\frac{3a}{5cT}\frac{g_{3/2}(z)}{g_{5/2}(z)} .$$

Since $c = ak$, the desired results follow readily.

Note that, as $z \to 1$, κ_T diverges in the same manner as γ and C_P.

(b) Since $P = 2U/3V$, $(\partial P/\partial T)_V = 2C_V/3V$. It follows that

$$C_P - C_V = TV \cdot \frac{1}{nkT}\frac{g_{1/2}(z)}{g_{3/2}(z)} \cdot \frac{4C_V^2}{9V^2} = \frac{4C_V^2}{9Nk}\frac{g_{1/2}(z)}{g_{3/2}(z)} ,$$

in agreement with eqn. (7.1.48a). The other result follows straightforwardly.

7.8. One readily sees that

$$w^2 = \left(\frac{\partial P}{\partial (mn)} \right)_s = \frac{1}{mn\kappa_s},$$

where κ_s is the adiabatic compressibility of the fluid. Using a result from Problem 7.5, we get for the ideal Bose gas

$$w^2 = \frac{5kT}{3m} \frac{g_{5/2}(z)}{g_{3/2}(z)} .$$

Next,

$$\langle u^2 \rangle = \left\langle \frac{2\varepsilon}{m} \right\rangle = \frac{2}{m} \frac{U}{N} = \frac{3kT}{m} \frac{g_{5/2}(z)}{g_{3/2}(z)} ;$$

see eqns. (7.1.8) and (7.1.11). Clearly, $w^2 = (5/9) < u^2 >$.

[redacted]

7.12. The relative mean-square fluctuation in N is given by the general formula (4.5.7),

$$\frac{\overline{(\Delta N)^2}}{\overline{N}^2} = \frac{kT}{V}\kappa_T , \tag{1}$$

while κ_T for the ideal Bose gas is given in Problem 7.5. As $T \to T_c$ from above, the function $g_{1/2}(z)$ and, along with it, both κ_T and the relative fluctuation in N diverge!

The mean-square fluctuation in E is given by the general formula (4.5.14), viz.

$$\overline{(\Delta E)^2} = kT^2 C_V + \left\{(\partial U / \partial N)_{T,V}\right\}^2 \overline{(\Delta N)^2} . \tag{2}$$

The first term in (2), for the ideal Bose gas, is determined by eqn. (7.1.37) and stays finite at *all* T. The second term can be evaluated with the help of eqns. (7.1.8 and 11), whereby

$$\left(\frac{\partial U}{\partial N}\right)_{T,V} = \left(\frac{\partial g_{5/2}(z)}{\partial g_{3/2}(z)}\right)_{T,V} = \frac{g_{3/2}(z)}{g_{1/2}(z)} . \tag{3}$$

The second term in (2) is, therefore, *inversely* proportional to $g_{1/2}(z)$ and hence vanishes as $T \to T_c$: this happens because the energy associated with the Bose condensate (which is, in fact, the component responsible for the dramatic rise in the fluctuation of N) is zero. Thus, all in all, the relative fluctuation in E is negligible at *all* T.

[redacted]

Pathria 7.18.

(a) The total work emits from sun is

$$W = A\sigma T^4 \simeq 4\times10^{26}\ W$$

the distance between sun and earth is 1.5×10^{11}m

at this distance, the energy is spread over the area $A = 4\pi R^2 \simeq 3\times10^{23}\ m^2$

$\Rightarrow$ the radiant intensity $I = \frac{W}{A} \simeq \frac{4}{3}\times10^3$

$= 1.33\times10^3\ W/m$

b) $I = \frac{1}{4}\frac{U}{V}c = \frac{1}{4}(3P)c = \frac{3cP}{4} \Rightarrow P = \frac{4I}{3c}$

$\because$ all photons are absorbed, there will be 1/2 factor

$\Rightarrow P = \frac{2I}{3c}$ here I we already got from part a)

$$P = \frac{2}{9\times10^8}\cdot\frac{4}{3}\times10^3 = \frac{8}{27}\times10^{-5}$$

c). if earth is a black body.

$$W_{out} = A\sigma T^4 = 4\pi R^2\sigma T^4 = W_{in} = I\cdot\pi R^2$$

$$\Rightarrow T = \left(\frac{I\pi R^2}{4\pi R^2\sigma}\right)^{1/4} = \left(\frac{I}{4\sigma}\right)^{1/4} = \left(\frac{\frac{4}{3}\times10^3}{4\times(5.67\times10^{-8})}\right)^{1/4}$$

$\simeq 280K \simeq 7°C$