5.13

From the first law $c_p = T\left(\frac{\partial s}{\partial T}\right)_p$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = T\left(\frac{\partial}{\partial p}\right)_T\left(\frac{\partial s}{\partial T}\right)_p = T\left(\frac{\partial}{\partial T}\right)_p\left(\frac{\partial s}{\partial p}\right)_T$$

Substituting the Maxwell relation $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$ and the definition $\alpha = -\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$ we have

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial}{\partial T}\right)_p \alpha v = \alpha^2 vT - vT\frac{d\alpha}{dT}$$

5.26

In the processes a→b and c → d no heat is absorbed , so by the first law $W = -\Delta E$, and since $\Delta E = C\Delta T$, where C is the heat capacity, we have

$$W_{a\to b} = -(E_b - E_a) = -\nu C_V\ (T_b - T_a)$$

$$W_{c\to d} = -(E_d - E_c) = -\nu C_V\ (T_d - T_c)$$

However, in an adiabatic expansion $TV^{\gamma-1}$ = const.

Hence
$$W_{a\to b} = -\nu C_V\ T_b\left(1 - \frac{T_a}{T_b}\right) = -\nu C_V\ T_b\left(1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}\right)$$

$$W_{c\to d} = -\nu C_V\ T_c\left(\frac{T_d}{T_c} - 1\right) = \nu C_V\ T_c\left(1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}\right)$$

The volume is constant in process b→c so no work is performed.

Thus
$$Q_1 = (E_c - E_b) = \nu C_V\ (T_c - T_b)$$

$$\eta = \frac{W_{a\to b} + W_{c\to d}}{Q_1} = 1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

8-3

$$P = C T^{5/2} f_{5/2}(z)$$

$$\left(\frac{\partial P}{\partial T}\right)_p = 0 = \frac{5}{2} C T^{3/2} f_{5/2}(z) + C T^{5/2} \frac{\partial f_{5/2}(z)}{\partial z} \left(\frac{\partial z}{\partial T}\right)_p .$$

$$\Rightarrow \quad \left(\frac{\partial z}{\partial T}\right)_p = -\frac{5}{2T} \frac{f_{5/2}}{\partial f_{5/2}/\partial z} .$$

$$\Rightarrow \quad \frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_p = -\frac{5}{2T} \frac{f_{5/2}}{f_{3/2}} .$$

$$\because \; C_p = T\left(\frac{\partial S}{\partial z}\right)_v \left(\frac{\partial z}{\partial T}\right)_p \quad \text{and} \quad C_v = T\left(\frac{\partial S}{\partial z}\right)_v \left(\frac{\partial z}{\partial T}\right)_v .$$

$$\gamma = \frac{C_p}{C_v} = \frac{(\partial z/\partial T)_p}{(\partial z/\partial T)_v} = \frac{5}{3} \frac{f_{5/2} f_{1/2}}{f_{3/2}^{2}} .$$

for High T. $\gamma = 5/3$

8.4 a) $K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ $\quad$ $K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_S$

$\because\ dV = \left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial V}{\partial T}\right)_P dT$ & $\left(\frac{\partial V}{\partial T}\right)_V = 0 = \left(\frac{\partial V}{\partial P}\right)_T\left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial V}{\partial T}\right)_P$

$$\Rightarrow \left(\frac{\partial V}{\partial P}\right)_T = -\frac{\left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial P}{\partial T}\right)_V} \quad \Rightarrow K_T = \frac{1}{V}\frac{\left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial P}{\partial T}\right)_V}$$

also $P = \frac{gkT}{\lambda^3} f_{5/2}$ with $\lambda = h(2\pi m k T)^{-1/2} = XT^{-1/2}$

$$\left(\frac{\partial P}{\partial T}\right)_V = gk\left(\frac{\partial}{\partial T}\frac{T^{5/2}}{X^3} f_{5/2}\right)$$ (using P.8.3 solution)

$$= \frac{n\lambda^3}{2gT}\left(\frac{5 f_{1/2} f_{5/2} - 3 f_{3/2}^2}{f_{3/2}^2}\right)$$

$$\Rightarrow K_T = \left(\frac{V}{Nk_T}\right)\left(\frac{f_{1/2}}{f_{3/2}}\right) = \frac{1}{nkT}\left(\frac{f_{1/2}}{f_{3/2}}\right).$$

for low T.

$$K_T = \frac{3}{2n}\left(1 - \frac{2\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2\right)\frac{1}{\varepsilon_F}\left(1 + \frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2\right)$$

$$= \frac{3}{2n\varepsilon_F}\left(1 - \frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2\right)$$

$$K_S = -\frac{1}{V}\left(\frac{\partial S}{\partial P}\right)_V\left(\frac{\partial S}{\partial V}\right)_P^{-1}$$

$$\left(\frac{\partial S}{\partial P}\right)_V = \left(\frac{\partial S}{\partial z}\right)_V\left(\frac{\partial z}{\partial T}\right)_V\left(\frac{\partial P}{\partial T}\right)_V^{-1}$$

$$\because \left(\frac{\partial S}{\partial z}\right)_{V,P} = \frac{Nk}{z}\left(\frac{3 f_{3/2}^2 - 5 f_{1/2} f_{5/2}}{f_{3/2}^2}\right)$$

$$\Rightarrow \left(\frac{\partial S}{\partial V}\right)_P = \frac{5gk f_{3/2}}{2\lambda^3} \quad \left[\left(\frac{\partial S}{\partial V}\right)_P = \left(\frac{\partial S}{\partial z}\right)_P\left(\frac{\partial z}{\partial T}\right)_P\left(\frac{\partial V}{\partial T}\right)_P^{-1}\right]$$

$$\Rightarrow \kappa_S = \frac{1}{V}\frac{\left(\frac{\partial S}{\partial p}\right)_V}{\left(\frac{\partial S}{\partial T}\right)_p} = \frac{3n}{5kT}\left(\frac{\lambda^3}{g}\right)^2 \frac{1}{f_{3/2} f_{5/2}}$$

$$\because \left(\frac{\lambda^3}{g}\right) = \frac{f_{3/2}}{n} \Rightarrow \kappa_S = \frac{3}{5nkT}\frac{f_{3/2}}{f_{5/2}}.$$

for low T $\quad \kappa_S = \frac{15}{2n\varepsilon_F}\left(1+\frac{\pi^2}{8}\left(\frac{kT}{\mu}\right)^2\right)$

b) $$\frac{C_p - C_v}{C_v} = \frac{T}{C_v}\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial V}{\partial T}\right)_p = \frac{4C_v}{9Nk}\frac{f_{1/2}}{f_{3/2}}.$$

$$\left(\because \left|\frac{5 f_{1/2} f_{5/2} - 3 f_{3/2}^2}{f_{1/2} f_{3/2}}\right| = \frac{4C_v}{3Nk}\right).$$

low T

$$\frac{C_p - C_v}{C_v} \simeq \frac{2kTC_v}{3Nk\varepsilon_F}\left(1-\frac{\pi^2}{6}\left(\frac{kT}{\varepsilon_F}\right)^2\right)\left(1+\frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_F}\right)^2\right)$$

$$\because \left(\frac{C_v}{Nk}\right) \simeq \frac{\pi^2 kT}{2\varepsilon_F} \Rightarrow \frac{C_p - C_v}{C_v} \simeq \frac{\pi^2}{3}\left(\frac{kT}{\varepsilon_F}\right)^2$$

c). $\gamma = \frac{\kappa_T}{\kappa_S} = \frac{5}{3}\frac{f_{1/2} f_{5/2}}{f_{3/2}^2}$ exact as the result from 8.3.