[redacted]

2. For a tricritical point, the only new exponent value is $\beta = 1/4$. Thus the Ginzburg criterion becomes:

$$\frac{\chi}{M^2 \xi^d} \ll 1 \Rightarrow t^{\gamma+2\beta-d\nu} \ll 1 \Rightarrow d > \frac{\gamma+2\beta}{\nu} = 3$$

* Note: Published series for $kT\chi$ give the 7^{th} order term as 1972. Does anyone see my error?

[redacted]

Series for triangular Ising ferromagnet:

a) Series for Z

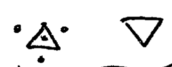
Following lecture it is relatively easy to compute this series to order v^6. At each order the allowed graphs and their degeneracies are:

v^0 - •

v^3 -  2N triangles on a lattice of N sites

v^4 -  3N graphs

v^5 -  6N graphs (there are 6 orientational possibilities)

v^6 -  + 

3N $\qquad$ $\frac{1}{2} \times 2N \times (2N-4)$

- 2N ways of placing 1st triangle, times
- 2N−4 " " " 2nd triangle − the 4 disallowed positions of the 2nd triangle are indicated by the heavy dot; these lead to shared bonds, times
- $\frac{1}{2}$ since the triangles are indistinguishable

so $Z \simeq 2^{N_S} (\cosh \beta J)^{N_B} \left[1 + 2Nv^3 + 3Nv^4 + 6Nv^5 + (3N + N(2N-4))v^6 + \dots \right]$

(b) Numerator in the series for χ

(i) Self avoiding walk configurations

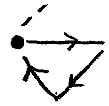
$n=1$ bond  6 configs

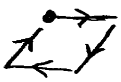
$n=2$  6×5 configs

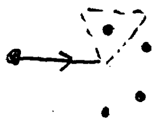
$n=3$  $(6 \times 5 \times 5$ − closed loops$) = 138$

The number of 3 bond loops is 12 − 6 directions for 1st step, 2 directions for 2nd, and 1 for the 3rd. To be more systematic, let c_n = number of n-step self avoiding walks, l_n = number of n-step walks which contain self intersection. In general,

$c_n = 6 \times 5^{n-1} - l_n$ and we now enumerate l_n.

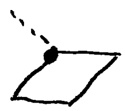
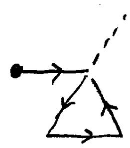
$\underline{n=4}$  60 12 triangles × 5 for the last bond

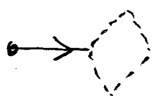
 24 6 directions for 1st step, 4 for 2nd, then last 2 steps are fixed

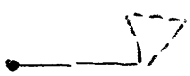
 48 6 directions for 1st step × 4 positions for trailing closed triangle (heavy dots for their centers) × 2 orientations in which triangle is traversed

$\Rightarrow l_4 = 132$ $\quad C_4 = 6 \times 5^3 - 132 = 618$

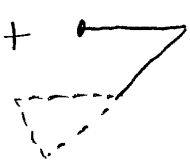
$\underline{n=5}$

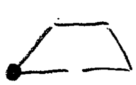
 300 +  120 +  240

+  84 = (6 ways for 1st bond × 7 distinct ways of attaching a quadrilateral × 2 orientations of traversal)

+  48 = (6 ways for 1st 2 bonds straight × 4 distinct ways of attaching a triangle × 2 orientations)

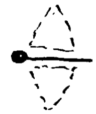
+  96 = (6 for 1st bond × 2 for 2nd bond × 4 triangles × 2 orientations)

+  72 = (6 for 1st bond × 2 for 2nd × 3 triangles × 2 orientations)

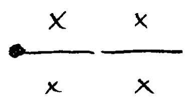
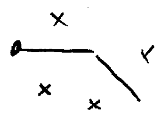
+  60

$l_5 = 1020$ $\quad \Rightarrow C_5 = 6 \times 5^4 - 1020 = 2730$

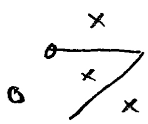
(ii) disconnected configurations

$n = 4$  $6(2N-2)$ = 6 directions for bond x
(2N-2) allowed possibilities for an additional triangle (disallowed positions of the triangle are indicated)

$n=5$  $6(3N-4)$ = 6 for bond x (3N-4) with 4 disallowed positions for a quadrilateral

+  + 

$18(2N-4)$ = 6 for 1st bond x 3 for 2nd bond x (2N-4) possibilities for an additional triangle (disallowed denoted by x

+ 

$12(2N-4)$

The last contribution is tricky - 6 for 1st bond x 2 for 2nd bond x (2N-4) for additional triangle : 3 disallowed possibilities are marked by the x ; the one marked by the circle is not allowed because the configuration  was already counted in the 1st two families of graphs in $l5$

Assembling everything :

$$kT\chi \cong 1 + A^{-1}\{6v + 30v^2 + 138v^3 + v^4(618 + 6(2N-2))$$
$$+ v^5(2730 + 6(3N-4) + 18(2N-4) + 12(2N-4)) + \cdots\}$$

$$A \simeq 1 + 2Nv^3 + 3Nv^4 + 6Nv^5 + [3N + N(2N-4)]v^6 + \cdots$$