MIDTERM EXAM **PY541** **October 25, 2005**

No notes or books are allowed or needed. Please be sure to **explain your work clearly** to maximize the probability of receiving partial credit.

1. A system consists of N non-interacting particles, each of which can be in one of two states, with respective energies ϵ_1 and $\epsilon_2 > \epsilon_1$.
   (a) Without performing an explicit calculation, (i) sketch the mean energy of the system as a function of temperature. Show explicitly the value of the mean energy in the (ii) low-temperature and in the (iii) high-temperature limits. (iv) Find the temperature at which the transition between these two limiting states occurs.
   (b) Sketch the heat capacity C_V as a function of temperature. Explain why this function has a single peak, and provide physical reasons for (i) the location of the peak, (ii) the height of the peak, and (iii) the width of the peak.

   For partial credit, work problem 1 by explicitly computing the partition function.

2. A heat engine runs between a finite size high-temperature reservoir of heat capacity C and an infinite lower-temperature reservoir. Initially the reservoir temperatures are T_{hot} and $T_{\text{cold}} < T_{\text{hot}}$. As a result of the engine operation, heat is extracted from the hotter reservoir until it reaches the temperature T_{cold}, after which the engine stops.
   (a) What is the total amount of heat extracted from the hotter reservoir?
   (b) Find the total entropy change of the finite reservoir during the engine operation.
   (c) Find the total entropy change of the system and thereby infer a lower bound on the heat produced by the engine exhaust.
   (d) Find the maximum work that can be produced by the engine. Is this work larger or smaller than the thermal energy initially stored in the reservoir? Explain.

3. An ideal Fermi gas consists of N spin-s particles with mass m in a box of volume V. Because of the particle spin, each quantum level has a degeneracy of $2s + 1$.
   (a) Find the Fermi energy of the system, ϵ_F (including the spin degeneracy factor).
   (b) Find the mean energy and the pressure at $T = 0$. How does the pressure depend on the particle density n and the spin degeneracy factor s? (If you can't do parts (a) and (b), determine how the Fermi energy and the mean energy depend on system parameters for partial credit.)
   (c) Numerically estimate the pressure exerted by the electrons in a typical metal. Compare this pressure with atmospheric pressure.
   (d) Suppose now that spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ Fermi gases are placed in separate compartments that are separated by a freely sliding wall. What is the relative density of the two gases in equilibrium at $T = 0$? What happens to the relative density as $T \to \infty$? (See sketch on the board.)