Assignment #4 PY 541 Week of Sept. 25–29, 2006

Reading: This week, we will treat the grand canonical ensemble and then give a short general discussion of the applications of thermodynamics. Please finish reading chapter 4 of the text. Reif, chapter 5 is a good and more comprehensive reference for this material.

Notes: I will be absent next Monday October 2 because of the Jewish holiday of Yom Kippur. Please feel free to schedule a time to see me either on the previous Friday or on Tuesday October 3.

Problems: Due Tuesday October 3.

1. Prove the equipartition theorem within the framework of classical statistical mechanics. Namely, for each quadratic term in either the generalized co-ordinates or momenta which appears in the Hamiltonian, there is a contribution of $\frac{1}{2}kT$ to the average energy of the system.

2. Text 3.14.

3. (Adapted from Plischke and Bergersen 2.6) Show that the grand canonical partition function of the ideal gas in a box of volume V is

$$\mathcal{Z} = \exp\left[e^{\beta\mu}\frac{V}{V_Q}\right]$$

   From this expression for $\mathcal{Z}$ compute the mean values of N, P, and S. Show that $PV = NkT$. Use the Gibbs-Duhem relation to derive $E = \frac{3}{2}NkT$. Finally show that $\langle(\Delta N)^2\rangle = N$ in the grand canonical ensemble.

4. (Reif 5.23) Two identical bodies with the same temperature-independent heat capacity C at constant pressure are used as heat reservoirs for a heat engine. The bodies remain at constant pressure and undergo no phase change. Initially, their temperatures are T_1 and $T_2 < T_1$. As a result of the heat engine operation, the bodies attain a common final temperature T_f.
   (a) What is the total work W done by the engine? Express your answer in terms of C, T_1, T_2, and T_f.
   (b) From entropic considerations, derive an inequality that relates T_f with T_1 and T_2.
   (c) For given initial temperatures T_1 and T_2, find the maximum work obtainable from the engine.