Assignment #10 PY 541 Week of Nov. 27 – Dec. 1, 2006

Reading: We discuss the Landau theory of phase transitions (Sec. 11.9 in the text) and then continue with the two-dimensional Ising model (Sec. 12.3). I will then (probably next week) discuss disordered spin systems, such as spin glasses and random-field models. For additional readings on these topics, I suggest "Equilibrium Statistical Mechanics" by M. Plischke and B. Bergersen, (Prentice Hall), and "Statistical Mechanics" by S.-K. Ma (World Scientific).

Notes: The final examination is scheduled for Wednesday Dec. 20 at 12:30pm in SCI 113. Please plan for a 3-hour exam. The exam be closed book, with no notes, calculators, or other ancillary material allowed. Considerable algebra should not be needed in solving any problem.

Problems: Due Friday Dec. 8 by 5pm. I assigned problem 3 last year in PY 541, so please don't consult with 2nd-year students.

1. Consider an Ising-like spin system on a hypercubic lattice in which the *magnitude* of the spin takes the values 1 and ϵ on alternating sites. Within the mean-field approximation, determine the critical temperature and the low temperature magnetization for both the "even" and "odd" sites. Comment on the behavior for small ϵ.

2. Consider the Bethe approximation for the Ising model, which represents a next approximation beyond the Curie-Weiss theory. For a lattice of coordination number z, consider a cluster of $z+1$ spins (in zero external magnetic field H) which consists of a central spin and its z surrounding nearest neighbors. Consider the ferromagnetic interaction between the central and boundary spins exactly, but assume that the interaction of the boundary spins with their nearest-neighbors can be replaced by an effective field of magnitude H'. Under these assumptions, compute the magnetization of the central and the boundary spins. Since these two magnetizations should be equal, this gives a self-consistency condition for H' which determines the critical temperature. Give the numerical value of the critical temperature (in units of J) for a lattice with $z = 4$.

3. Consider a ferromagnetic Ising model on the complete graph in which N sites are all interconnected by bonds. (For example, a 4-site complete graph is a tetrahedron with 6 sides. A general N-site complete graph has $\binom{N}{2}$ bonds). Further, suppose that the interaction strength of the Ising model on such a complete graph is $j = J/N$, with J independent of N. (This is chosen so that the system has a finite energy per spin in the thermodynamic limit). (a) Without detailed computation, determine whether there is a spontaneous magnetization in this system at sufficiently low, but non-zero temperature. (b) Compute the partition function of this Ising model, in zero and finite magnetic field. (c) Compute the spontaneous magnetization.