Assignment #1   PY541   Week of Sept. 4–8, 2006

Reading: Please read sections 1.1 - 1.3, 2.1 - 2.5, and 14.3 in Pathria. For additional information about the binomial distribution and its continuum limit, please consult the beginning chapters in Reif and/or Kittel and Kroemer.

Problems: Due Monday September 11 by 5pm in Pu Chen's mailbox.

1. (Reif 1.9 & 1.11) Consider a binomial process for the probability of n events in N trials, $P_N(n)$, in which the probability p for an event in a single trial is much less than 1. Show that in this limit, $P_N(n) \approx \mu^n e^{-\mu}/n!$, with $\mu = Np$. This form is known as the *Poisson* distribution. Consider now a 600-page book in which there are 600 randomly-located typographical errors. From the Poisson distribution, find that probability that a page contains (i) no errors, and (ii) at least 3 errors.

2. (Reif 1.15) A set of telephone lines is to be installed to connect town A, with 2000 telephones, to town B. Suppose that during the busiest hour of the day, each subscriber in A requires, on average, a two-minutes telephone connection to B. Assume that these calls are made at random. Find that minimum number of telephone lines to B so that at most 1% of the callers from A will fail to have immediate telephone access to B. (Hint: Use the Gaussian approximation for the distributions in this problem).

3. (a) Derive the $N \to \infty$ (continuum) limit of the binomial process for general $p \neq q$. (b) You are playing a coin tossing game in which you win \$1 if a head appears, while you lose \$1 if a tail appears. After 10000 trials, you've lost \$300. You suspect that the game is being played with a biased coin. If the coin is biased, you know that tails occurs with a probability of 0.51. What is the probability that the game is being played with a biased coin? What test could you devise to be 90% certain that the coin is biased?

4. Consider an isotropic one-dimensional random walk in which at the nth step a hop of length n to the left or length n to the right occurs, each with probability 1/2. Following the same steps as those used in the discussion of the central limit theorem, find the continuum $N \to \infty$ form of $P_N(x)$. Can you give a physical argument for the relation between the mean-square displacement and the number of steps?