Reading: Please read sections 6.1 and 6.2 in the text. We will finish discussing non-degenerate perturbation theory this week (section 6.1) and will probably begin treating degenerate perturbation theory (section 6.2) sometime on Thursday.

1. This problem is based mostly on problem 6.1 of the text. This problem asks for the perturbed energies and eigenfunction in an infinite square-well potential in $[0, a]$ with an additional delta-function spike in the middle of the potential. In addition, please answer the following extensions of the original problem:
   
   (a) Determine the first-order corrections to the energies levels and to the ground-state wavefunction when the perturbation is $H' = \alpha \delta(x - b)$; that is, the spike is not in the middle of the potential well.
   
   (b) Determine the first-order corrections to the first two energy levels and to the wavefunction of the ground and first excited states when the perturbation is a small square bump. That is $H' = V_0$ for $|x - a/2| < b$, (and $b < a/2$ so that the bump does not extend past the walls of the potential well) and $H' = 0$ otherwise.

2. Griffiths 6.4(a). The problem asks for the second-order corrections for the infinite square-well potential with an additional delta-function spike.

3. Calculate the energy shift in the ground state of the one-dimensional harmonic oscillator with an additional anharmonic term that is proportional to $x^4$. That is, the Hamiltonian is
   
   $$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} + \lambda x^4.$$  

   Redo the problem with the perturbation term is $\lambda x^6$.

4. Calculate, to first order, the change in the first three energy levels and the first three energy eigenstates in the infinite square-well potential on $[0, a]$ with the additional perturbation $\delta V(x) = \lambda x/a$. Calculate, to first order, the average position of the particle in the ground state.