

**PY 451: Quantum Physics I Problem Set 9**

**This Week:** This week will be devoted to spin angular momentum. The two major topics that we’ll cover are the dynamics of an electron in a magnetic field and the addition of angular momenta.

**Reading for the week:** Section 4.4 in the text.

**Problems:** due Friday, April 15 by 5:00pm.

1. For the spinor $\frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, calculate the probability that a measurement of $(3S_x + 4S_y)/5$ yields the value $-\hbar/2$.

2. Using the algebra of the raising and lowering operators, compute the spin matrices $S_x$, $S_y$, and $S_z$ for a spin-$\frac{3}{2}$ particle.

3. Show that any $2 \times 2$ matrix $M$ may be written in the form

   $$M = A + B \cdot \sigma,$$

   where $\sigma$ is the vector whose elements are the Pauli matrices and $B$ is some given vector. Determine the conditions on $A$ and $B$ such that (i) $M$ is unitary, and (ii) $M$ is hermitian.

4. A spin-$\frac{1}{2}$ electron is in an eigenstate of $S_x$ with eigenvalue $\hbar/2$ at time $t = 0$. At this time, the electron is placed in a magnetic field of magnitude $B$ that points in the $z$-direction for a time $T$. At time $T$, the direction of the field is suddenly changed so that it points in the $y$-direction over the time range $[T, 2T]$. At time $t = 2T$ a measurement of $S_x$ is performed. Calculated the probability that this measurement gives the value $\hbar/2$. 