Announcements: This week we will finish the discussion of the underlying formalism of quantum mechanics and then continue to quantum mechanics in greater than one dimension.

Reminder: For the longer term, the second midterm will be given during the lecture period on Tuesday April 5. In a few weeks, I’ll post a copy of the second midterm from last year on the course website as a study aid.

Reading for March 7–11: Please finish reading chapter 3 of the text and also start reading section 4.1 of the text.

Problems: due Friday, March 11 by 5:00pm.

1. Given a Hermitian operator $H$ with the property that $H^4 = \mathbb{1}$, where $\mathbb{1}$ is the unit operator. What are the possible eigenvalues of $H$? What are the eigenvalues of $H$ if the operator is not Hermitian?

2. Prove that the Hermitian conjugate of the Hermitian matrix $AB$, that is, $(AB)^\dagger$, is $B^\dagger A^\dagger$ (note the order). This same statement also applies to Hermitian operators. Next, show that if $H$ is a Hermitian, then $(e^{iH})^\dagger = e^{-iH}$. Finally, an operator $U$ is unitary is $UU^\dagger = U^\dagger U = 1$. Show that if $H$ is Hermitian, then $e^{iH}$ is unitary.

3. Use the formalism developed in lecture to find the equations of motion for the expectation values of $\langle x \rangle$ and $\langle p \rangle$ for the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2} m(\omega_1^2 x^2 + \omega_2 x + C),$$

where $C$ is some constant. For extra credit, solve these equations of motion for an arbitrary initial condition.

4. By expanding the exponentials in power series, verify that the first 4 series terms for the operator $e^A B e^{-A}$ are:

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2!} [A, [A, B]] + \frac{1}{3!} [A, [A, [A, B]]] + \ldots$$

5. Consider the Hamiltonian for a one-dimensional oscillator in an external electric field in the Heisenberg picture:

$$H = \frac{p(t)^2}{2m} + \frac{1}{2} m\omega^2 x(t)^2 - eE x(t)$$

Using the formalism discussed in lecture, calculate the equations of motion for the operators $p(t)$ and $x(t)$. You may make use of the canonical commutation relation $[x(t), p(t)] = i\hbar$. The result should be the classical equations of motion. Solve for $p(t)$ and $x(t)$ in terms of $p(0)$ and $x(0)$. Finally, show that $[x(t_1), x(t_2)] \neq 0$ for $t_1 \neq t_2$. 