

# Solns HW 8

1)  $\Psi_{n_1, n_2, n_3}(x, y, z) = U_{n_1}(x) U_{n_2}(y) U_{n_3}(z)$

$U_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad n = 1, 2, 3, \dots$

$E = E_{n_1} + E_{n_2} + E_{n_3}$  in units of  $\frac{\hbar^2 \pi^2}{2ma^2}$

(111)	3	1
(112)	6	3
(122)	9	3
(113)	11	3
(222)	12	1
etc		

2)  $E = \hbar\omega(n_1 + n_2 + n_3 + \frac{3}{2}) \quad n = 0, 1, 2, 3, \dots$

(000)	3/2	1
(100)	5/2	3
(110)	7/2	6
(200)		

etc

$$3) \quad \frac{x}{r} = \sin\theta \cos\phi \quad \frac{y}{r} = \sin\theta \sin\phi \quad \frac{z}{r} = \cos\theta$$

$$Y_{1\pm 1} = \mp \sqrt{\frac{3}{8\pi}} \sin\theta e^{\pm i\phi} = \mp \sqrt{\frac{3}{8\pi}} \frac{x \pm iy}{r}$$

$$Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta = \sqrt{\frac{3}{4\pi}} \frac{z}{r}$$

$$Y_{22} = \sqrt{\frac{15}{32\pi}} e^{2i\phi} \sin^2\theta = \sqrt{\frac{15}{32\pi}} \left( \frac{x^2 - y^2 + 2ixy}{r^2} \right)$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}} e^{i\phi} \sin\theta \cos\theta = -\sqrt{\frac{15}{8\pi}} \frac{(x+iy)z}{r^2}$$

$$Y_{20} = \sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1) = \sqrt{\frac{5}{16\pi}} \left( \frac{2z^2 - x^2 - y^2}{r^2} \right)$$

$$Y_{l(-m)} = (-1)^m Y_{lm}^*$$

$$4) \quad [x, L_x] = [x, yP_z - zP_y] = 0$$

$$[y, L_x] = [y, yP_z - zP_y] = -i\hbar z$$

$$[z, L_x] = [z, yP_z - zP_y] = i\hbar y$$

The above is cyclic in  $(x, y) \rightarrow z$