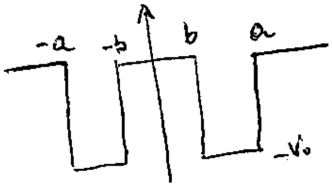
a) 

by symmetry we only need to match wave functions at $x=b$ and a.

For <u>even</u> solutions we have

$$\begin{cases} \phi_1(x) = A\cosh \kappa x & 0<x<b \\ \phi_2(x) = B\sin qx + C\cos qx & b<x<a \\ \phi_3(x) = De^{-\kappa x} & x>a \end{cases}$$

<u>at $x=b$</u> $\quad \phi_1(b)=\phi_2(b) \rightarrow A\cosh\kappa b = B\sin qb + C\cos qb$

$\quad \phi_1'(b)=\phi_2'(b) \rightarrow \kappa A\sinh\kappa b = q(B\cos qb - C\sin qb)$

<u>at $x=a$</u> $\quad \phi_2(a)=\phi_3(a) \rightarrow C\cos qa + B\sin qa = De^{-\kappa a}$

$\quad \phi_2'(a)=\phi_3'(a) \rightarrow q(B\cos qa - C\sin qa) = -\kappa De^{-\kappa a}$

After some manipulation one gets the result.

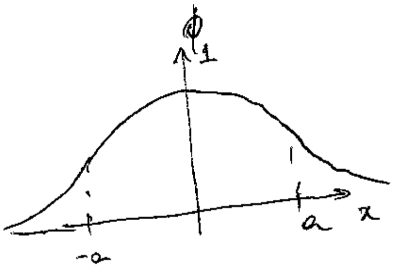
For <u>odd</u> solutions the only difference is $\underline{\phi_1(x) = A\sinh\kappa x}$.

b) as $b\to 0$ $\quad t(q(a-b)) \to tqa$ $\quad$ so for <u>even sol.</u> $\tan qa = \frac{\kappa}{q}$

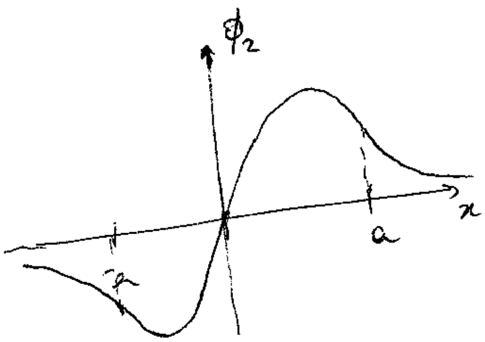
$\tanh\kappa b \to 0$

$\coth\kappa b \to +\infty$

and for <u>odd sol.</u> $\tan qa = -q/\kappa$

c) 

ground st.

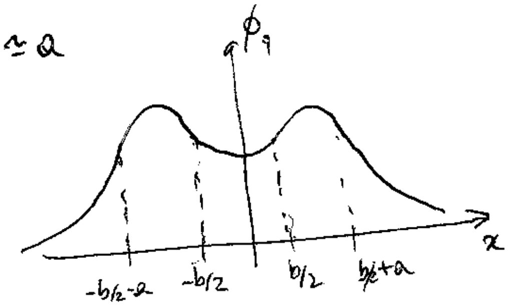


1st exc. state

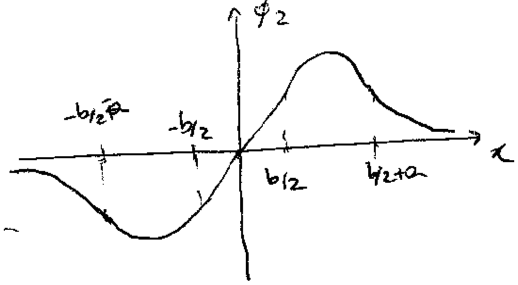
$b=0$

normal finite square well

$b \simeq a$



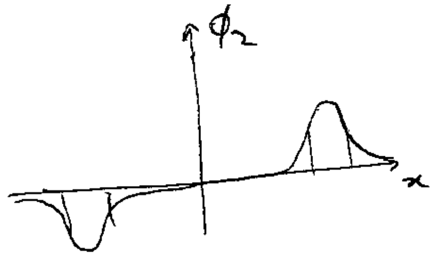
ground state



1st exc. state

the solution is given in part $\underline{a}$

$\underline{b \gg a}$ is same as $b \approx a$ but the wave function is very small in the barrier region. as $b \to \infty$ we have two isolated wells, with a degenerate ground state.



The energy of ϕ_1 becomes equal to $E(\phi_2)$ as $b \to \infty$

d) for $b=0$

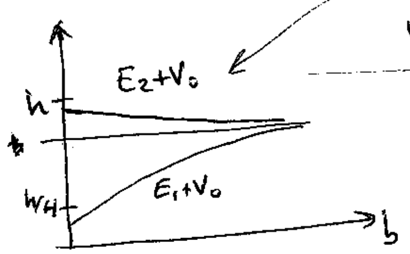
$$\begin{cases} E_1+V_0 \simeq \dfrac{\pi^2\hbar^2}{2m(2a)^2} = \dfrac{h}{4} \\ E_2+V_0 \simeq h \end{cases}$$

with $h = \dfrac{\pi^2\hbar^2}{2ma^2}$

$b \to \infty$ (isolated wells)

$$\left\{ E_1+V_0 = E_2+V_0 \simeq \frac{\pi^2\hbar^2}{2ma^2} = h \right.$$

As $b \to 0$ ϕ_2 gets more curved, corresponding with higher energy



e) In the ground state the energy is lowered by reducing $\underline{b}$ so the electron tends to draw the nuclei <u>together</u>.

The opposite happens in the 1st exc. state.