Solutions to HW 4

1) An arbitrary soln to the Schrödinger Eqn for the infinite square well is:

$$\Psi(x,t) = \sum_{n=1}^{\infty} C_n \psi_n(x)\, e^{-i\frac{n^2\pi^2\hbar}{2ma^2}t}$$

Thus by replacing t w/ the revival time T it is trivial to show

$$\Psi(x,0) = \Psi(x,T)$$

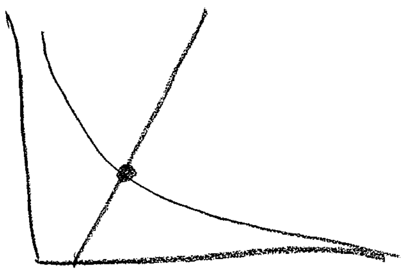
For a classical particle the period of movement is $T_c = \frac{2a}{v}$ where v is defined by the Kinetic Energy. Thus $T_c = a\sqrt{\frac{2m}{E}}$.

For the times to be equal

$$E_c = \frac{E_{n=1}}{4}$$

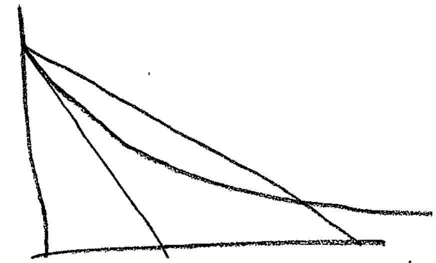
Solving these 2 Eqns graphically

Even



Aways 1 soln

odd



up to 1 soln

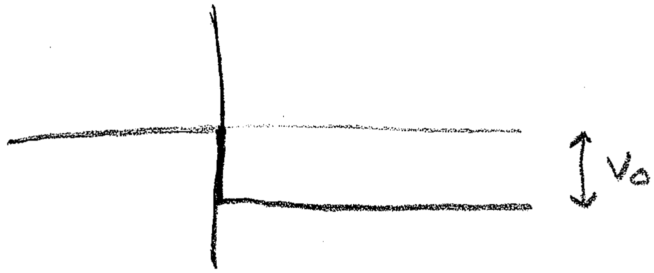
for $\alpha = \frac{\hbar^2}{ma}$

$$E = -0.615\,\frac{\hbar^2}{ma^2}\ ; \quad E = -0.317\,\frac{\hbar^2}{ma^2}$$

for $\alpha = \frac{\hbar^2}{4ma}$ only an even soln exists

$$E = -0.0682\,\frac{\hbar^2}{ma}$$

4)



$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < 0 \\ Fe^{ilx} & x > 0 \end{cases}$$

w/ $k \equiv \dfrac{\sqrt{2mE}}{\hbar}$ $\quad l \equiv \dfrac{\sqrt{2m(E+V_0)}}{\hbar}$

From continuity of ψ, ψ'

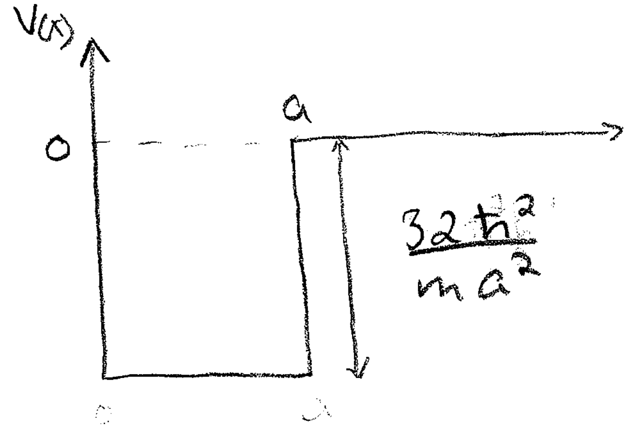
$$A + B = \frac{k}{l}(A - B) \Rightarrow \frac{B}{A} = -\left(\frac{1 - k/l}{1 + k/l}\right)$$

$$R = \left|\frac{B}{A}\right|^2 = \left(\frac{\sqrt{E+V_o} - \sqrt{E}}{\sqrt{E+V_o} + \sqrt{E}}\right)^2$$

Thus for $E = 4\,eV$, $V_o = 12\,eV$

$$T = 1 - R \Rightarrow T = \frac{8}{9}$$

5)



w/ $V_0 \equiv \frac{32\hbar^2}{2ma^2}$ this is only the odd states of the symmetric finite well of same height.

$$\psi(x) = \begin{cases} D \sin lx \\ F e^{-\kappa x} \end{cases} \qquad l \equiv \frac{\sqrt{2m(E+V_0)}}{\hbar} \text{ inside} \qquad \kappa \equiv \frac{\sqrt{-2mE}}{\hbar} \quad x > a$$

this yeilds $-\cot z = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1}$

$$z_0 = \frac{\sqrt{2mV_0}}{\hbar} a = 8$$

Therefor, Solving Graphically yeilds 3 bound States!

By matching BC's

$$P = \frac{z^2}{z_0^2\left(1+\sqrt{z_0^2-z^2}\right)} \quad \text{or} \quad .54204$$