(1) $E_n = \dfrac{\hbar^2 \pi^2 n^2}{2ma^2} \rightarrow \boxed{E_2 - E_1 = \dfrac{3}{2}\dfrac{\hbar^2\pi^2}{ma^2} = 1,14\ eV} \rightarrow \boxed{\lambda = \dfrac{hc}{\Delta E} = 1,44 \times 10^{-6}\, m}$

(2) The wave function does not remain localized at $t>0$, since the phase relationship that allows for a total interference no longer persists for $t \neq 0$

~~[illegible]~~ since a shift has no physical consequences the solution is the same as having the box extending from $x=0$ to $x=a$

$$C_n = \int_0^a \psi(x)\, u_n(x)\, dx = \frac{2}{a}\int_0^{a/2} \sin\left(\frac{n\pi}{a}x\right) dx = \frac{2}{n\pi}\left[1 - \cos\frac{n\pi}{2}\right]$$

$u_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$

$$P_1 = |C_1|^2 = |C_2|^2 = P_2 = \frac{4}{\pi^2}$$

(3) initially $\psi(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{\pi}{a}x\right)$ (ground state prob. 2)

$-a/2 < x < a/2$

i) $P_1 = |C_1|^2$ with $C_1 = \int \psi(x)\, u_1(x)\, dx$ $\qquad u_1 = \sqrt{\frac{1}{b}} \cos\left(\frac{\pi}{2b}x\right)$

$$\boxed{C_1 = \sqrt{\frac{2}{ab}} \int_{-a/2}^{a/2} dx \cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi x}{2b}\right) = \frac{\sqrt{2ab^3}}{\pi(4b^2 - a^2)} \cos\left(\frac{\pi a}{4b}\right)}$$

ii) $P_2 = |C_2|^2 \qquad C_2 = \int_{-a/2}^{a/2} \underbrace{\psi(x)}_{\text{even}}\underbrace{u_2(x)}_{\text{odd}}\, dx = 0 \rightarrow \boxed{P_2 = 0}$

(4) $\phi(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(x)\, e^{-ipx/\hbar} = \left(\frac{1}{\pi\alpha\hbar^2}\right)^{1/4} e^{-p^2/2\alpha\hbar^2}$

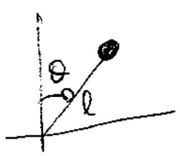
Probability of p being between p and $p+dp$ is $|\phi(p)|^2 dp$

$$E = \frac{p^2}{2m} \rightarrow \langle E \rangle = \frac{1}{2m}\langle p^2 \rangle = \frac{1}{2m}\left(\frac{1}{\pi\alpha\hbar^2}\right)^{1/2} \int_{-\infty}^{+\infty} dp\, p^2 e^{-p^2/\alpha\hbar^2} = \frac{\alpha\hbar^2}{2m}$$

$\Delta x \simeq \frac{1}{\sqrt{\alpha}}$ from uncertainty pple $\Delta p \simeq \hbar/\Delta x = \hbar\sqrt{\alpha}$

so $E \simeq \frac{(\Delta p)^2}{2m} = \frac{\alpha\hbar^2}{2m}$

(5)



initial angle θ_0, initial angular momentum L_0

pencil = mass at end of a massless stick

$$\Delta x . \Delta p \simeq \hbar \rightarrow l\theta_0 . l m \omega_0 \simeq \hbar \rightarrow \theta_0\omega_0 \simeq \frac{\hbar}{ml^2}$$

$$ml\ddot{\theta} = mg\sin\theta \rightarrow \ddot{\theta} = g/l \sin\theta \simeq g/l\, \theta$$ (only valid for small θ but good for an estimation)

$$\rightarrow \theta(t) = A e^{-t/\tau} + B e^{t/\tau} \quad \text{wit } \tau = \sqrt{l/g}$$

initial cond. $\begin{array}{l} A+B = \theta_0 \\ A-B = \omega_0\tau \end{array} \rightarrow B = \frac{\theta_0 + \omega_0\tau}{2}$

$$\theta(t^*) = \frac{\pi}{2} \simeq B e^{t^*/\tau} \quad (\text{assuming } t^* \gg \tau) \rightarrow t^* \simeq \tau \ln\left(\frac{\pi}{\theta_0 + \omega_0\tau}\right)$$

$$t^* \simeq \tau \ln\left(\frac{\pi\theta_0}{\theta_0^2 + \theta_0\omega_0\tau}\right) = \tau \ln\left(\frac{\pi\theta_0}{\theta_0^2 + \underbrace{\frac{\hbar}{m\sqrt{gl^3}}}_{=\alpha^2}}\right) = \tau \ln\left[\frac{\pi}{\alpha}\, \frac{\theta_0/\alpha}{(\theta_0/\alpha)^2 + 1}\right]$$ maximum for $\theta_0 \simeq \alpha$

$$t^* \simeq \tau \ln \alpha^{-1}$$

$$\alpha^2 = \frac{\hbar}{m\sqrt{gl^3}} = \frac{10^{-34}}{10^{-3}(10 . 10^{-6})^{1/2}} \simeq 10^{-29} \rightarrow \alpha \simeq 10^{-29/2} \rightarrow \boxed{t^* \simeq \sqrt{\frac{l}{g}} \ln 10^{\frac{29}{2}} \simeq 30\sqrt{\frac{l}{g}}}$$

$t^* \gg \tau$ ✓