Solutions to HW 2:

PY 451:

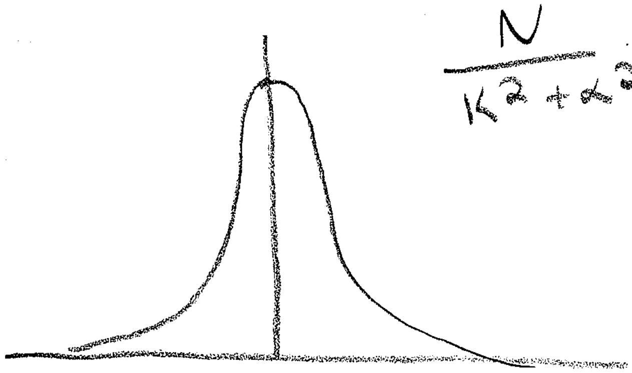
1) We Know $2\psi^s(x) = \int_{-\infty}^{\infty} A(k)\, e^{ikx}\, dx$

Since only the even portion of e^{ikx} will contribute to the integration:

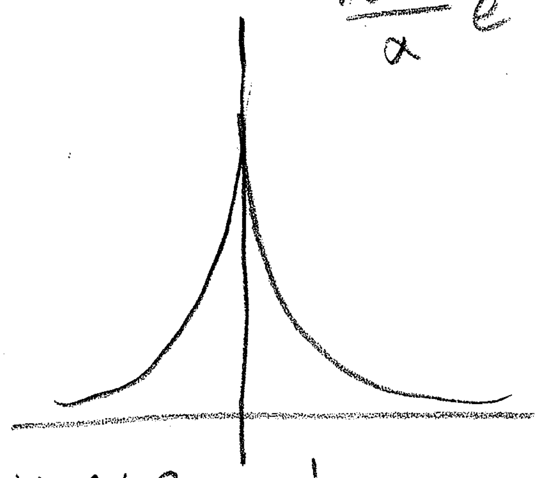
$$2\psi^s(x) = \int_{-\infty}^{\infty} \frac{N}{k^2+\alpha^2} \cos kx \, dk = N \frac{\pi}{\alpha} e^{-\alpha|x|}$$

$$\therefore\ 2\psi^{s2}(x) = \frac{N^2\pi^2}{\alpha^2} e^{-2\alpha|x|}$$

$\frac{N}{k^2+\alpha^2}$



$\frac{N\pi}{\alpha} e^{-\alpha|x|}$



by examining plots $\Delta k \simeq 2\alpha$ & $\Delta x \simeq \frac{1}{2\alpha}$

This gives the expected $\Delta k \Delta x \simeq 1$

2) With $w(t) = \frac{\hbar K^2}{2m}$, $\beta = \hbar/m$

& $w(0) = \sqrt{2}\,\alpha$

Thus $$\frac{w(t)}{w(0)} = \sqrt{1 + \frac{\beta^2 t^2}{2\alpha^2}} = \sqrt{1 + \frac{2\hbar^2 t^2}{m^2 w^4(0)}}$$

So w/ $\underline{t=1}$ & $w(0) = 10^{-6}$

$w(1) = 1.7 \times 10^{2}\,m$

w/ $w(0) = 10^{-10}$

$w(1) = 1.7 \times 10^{6}$

b) for $m = 10^{-3}\,Kg$ & $w(0) = 10^{-2}\,m$

$$\frac{2\hbar^2 t^2}{m^2 w^4(0)} \doteq 2.2 \times 10^{-54}$$

which is negligable wrt $\underline{1}$.

3) The momentum-space wave-fn is given by the Fourier transform:

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dx \; \psi(x)\, e^{-ipx/\hbar} \qquad \text{let } K = \frac{p}{\hbar}$$

$$= \frac{A}{\sqrt{2\pi\hbar}} \int_{-\infty}^{0} dx\, e^{\mu - iKx} + \int_{0}^{\infty} e^{\mu + iKx}\, dx$$

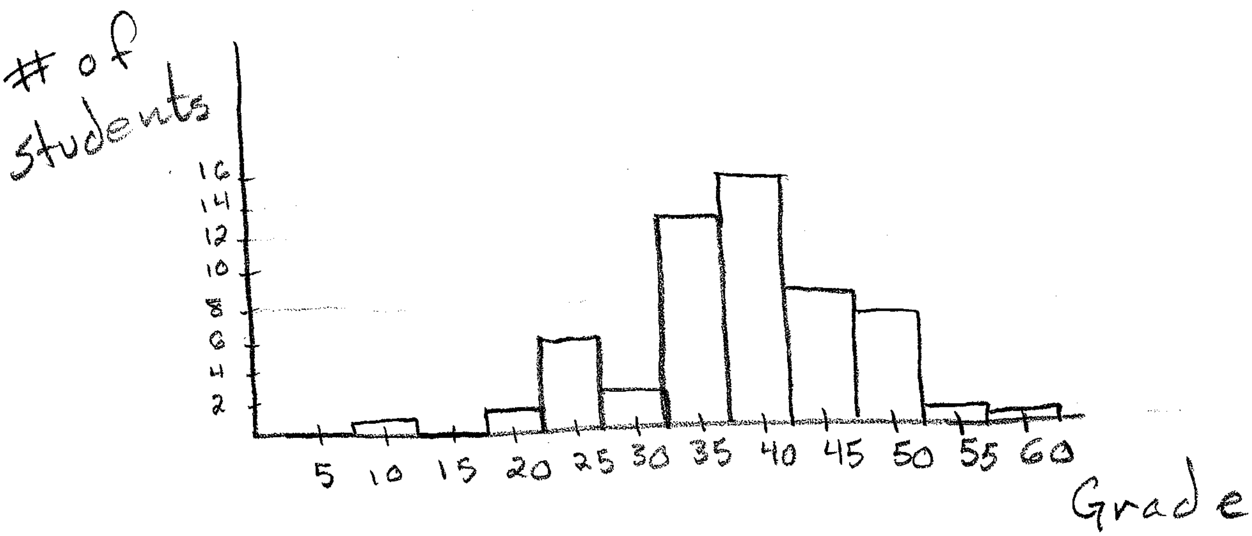
$$= \frac{A}{\sqrt{2\pi\hbar}} \frac{2\mu}{\mu^2 + K^2}$$

4)

$$\langle g \rangle = \sum_g g n_g = 38.5$$

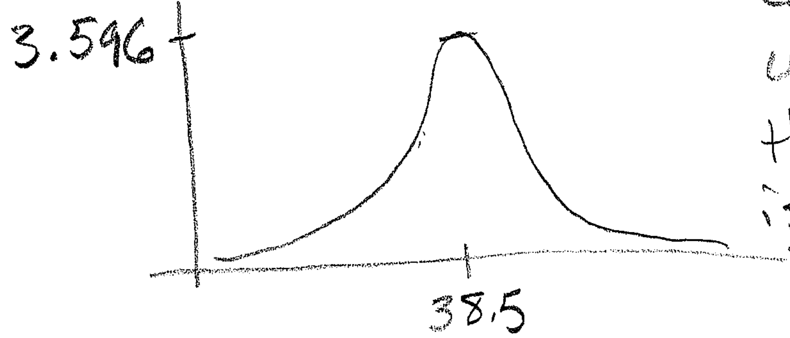
$$(\Delta g)^2 = \langle g^2 \rangle - \langle g \rangle^2 = \sum_g g^2 n_g - (38.5)^2$$

$$= 88.6$$



Now create, $N(g) = C e^{-(g - \langle g \rangle)^2 / \sigma^2}$

w/ $\int_0^\infty N(g)\, dg = 60$ then $C \cong 3.596$



Calibrating s.t. the area under = 60 is <u>not</u> the same since $N(g)$ is a continuous Distribution!

5) We know $\langle x^n \rangle = \int_{-\infty}^{\infty} \psi^*_{(x)} x^n \psi_{(x)} dx$

w/ $\psi_{(x)} = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$

Thus $\langle x^1 \rangle = 0 = \langle x^{17} \rangle$ b/c the integrand is even over an odd interval.

$$\langle x^2 \rangle = \left(\frac{\alpha}{\pi}\right)^{1/2} \int_{-\infty}^{\infty} e^{-\alpha x^2} x^2 dx = \left(\frac{\alpha}{\pi}\right)^{1/2} \frac{1}{2} \frac{\sqrt{\pi}}{a^{3/2}}$$

$$\langle x^2 \rangle = \frac{1}{2a}$$

$$\phi(p) = \left(\frac{\alpha}{\pi}\right)^{1/4} \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar} e^{-\alpha x^2/2} dx$$

by completing the square in the exponential the integral maybe reduced to a gaussian:

$$\phi(p) = \left(\frac{1}{\pi \alpha \hbar^2}\right)^{1/4} e^{-\frac{p^2}{2\alpha\hbar^2}}$$

5) con't

Thus, again $\langle p' \rangle$ will be an odd integrand over an even inteval

$\Rightarrow \langle p' \rangle = 0$

$$\langle p^2 \rangle = \left(\frac{1}{\pi \alpha \hbar^2}\right)^{1/2} \int_{-\infty}^{\infty} p^2 e^{-\frac{p^2}{\alpha \hbar^2}}$$

$$= \left(\frac{1}{\pi \alpha \hbar^2}\right)^{1/2} \frac{1}{2} \hbar^3 a^{3/2} \sqrt{\pi}$$

$$= \frac{\hbar^2 a}{2}$$