HW 12 Solns

1) a) Adding 3 spin 1/2's

$\frac{3}{2}$ or $\frac{1}{2}$

b) Adding 2 spin 1/2's

0 or 1

4) $\vec{J}\cdot\vec{J} = \vec{L}\cdot\vec{L} + \vec{S}\cdot\vec{S} + 2\vec{L}\cdot\vec{S}$

$$\vec{L}\cdot\vec{S} = \frac{\vec{J}\cdot\vec{J} - \vec{L}\cdot\vec{L} - \vec{S}\cdot\vec{S}}{2} = \frac{j(j+1) - \ell(\ell+1) - s(s+1)}{2}$$

$$= \frac{1}{2}\left[j(j+1) - \ell(\ell+1) - 2\right]$$

$j = \ell - 1 \Rightarrow \vec{S}\cdot\vec{L} = -\ell - 1$

$j = \ell + 1 \Rightarrow \vec{S}\cdot\vec{L} = +\ell$

$j = \ell \Rightarrow \vec{S}\cdot\vec{L} = -1$

$$V(j=\ell-1) = V_1 - \frac{\ell+1}{\hbar^2}V_2 + \frac{(\ell+1)^2}{\hbar^2}V_3$$

$$V(j=\ell+1) = V_1 + \ell V_2 + \ell^2 V_3$$

$$V(j=\ell) = V_1 - V_2 + V_3$$

2) $\chi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad \chi_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\quad \chi_{-1} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$S_z \chi_j = j \hbar \chi_j \longrightarrow S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \hbar$$

$\cancel{S_\pm \chi_j} \Rightarrow$

$$S_+ \chi_1 = 0 \qquad S_- \chi_{-1} = 0$$
$$S_+ \chi_0 = \sqrt{2}\hbar \chi_1 \qquad S_- \chi_1 = \sqrt{2}\hbar \chi_0$$
$$S_+ \chi_{-1} = \sqrt{2}\hbar \chi_0 \qquad S_- \chi_0 = \sqrt{2}\hbar \chi_{-1}$$

$$S_+ = \sqrt{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad S_- = \sqrt{2}\hbar \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S_\pm = S_x \pm i S_y \longrightarrow S_x = \frac{S_+ + S_-}{2} \qquad S_y = \frac{S_+ - S_-}{2i}$$

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

3) ~~...~~ since $J_z = J_z^1 + J_z^2 \rightarrow m = m_1 + m_2$

$[J, J_z^1] = [J, J_z^2] = 0$

$m_1 = 1, 0$ or $-1 \rightarrow m_2 = 2, 1,$ or 0

$|31\rangle = c_1 |11\rangle|20\rangle + c_2 |10\rangle|21\rangle + c_3 |1-1\rangle|22\rangle$

we know that

$|33\rangle = |11\rangle|22\rangle$ and $S_- = S_-^1 + S_-^2$

$S_-|33\rangle = \sqrt{6}\,|32\rangle = S_-^1|11\rangle|22\rangle + |11\rangle S_-^2|22\rangle$

$\rightarrow |32\rangle = \sqrt{2/6}\,|10\rangle|22\rangle + \frac{2}{\sqrt{6}}|11\rangle|21\rangle$

applying S_- again $S_-|32\rangle = \sqrt{10}\,|31\rangle = (\quad)$

$\rightarrow |31\rangle = \sqrt{2/5}\,|11\rangle|20\rangle + \sqrt{\frac{8}{15}}\,|10\rangle|21\rangle + \frac{1}{\sqrt{15}}|1-1\rangle|22\rangle$