# PY 451: Quantum Physics I  Problem Set 9

Due date: Friday, April 3 2009, by 5:00pm

1. Use separation of variables to solve the problem of a particle inside an infinite circular potential well in *two dimensions*; that is $V(\mathbf{r}) = 0$ for $r < a$ and $V(\mathbf{r}) = \infty$ for $r > a$. Specifically, write the elemental solutions (the eigenfunctions) for the radial and angular wavefunctions, and finally, write the general solution as a sum over the eigenfunctions.

2. (text 8-10) An electron that moves in the Coulomb field of a proton is prepared in a state that is described by the wave function

$$\frac{1}{6}\left[4\psi_{100}(\mathbf{r}) + 3\psi_{211}(\mathbf{r}) - \psi_{210}(\mathbf{r}) + \sqrt{10}\,\psi_{21-1}(\mathbf{r})\right]$$

where the subscripts on $\psi$ refer to the quantum numbers $n, \ell, m$.

   (a) What is the expectation value of the energy?

   (b) What is the expectation value of $\mathbf{L}^2$?

   (c) What is the expectation value of $L_z$?

3. (based on text 8-12 & 9) A basic fact is that the expectation value of an arbitrary function of the form $f(\mathbf{r}, \mathbf{p})$ in any stationary state is a constant. Using this result, calculate

$$\frac{d}{dt}\langle \mathbf{r}\cdot\mathbf{p}\rangle = \frac{i}{\hbar}\langle [H, \mathbf{r}\cdot\mathbf{p}]\rangle$$

for a Hamiltonian of the form $H = \frac{\mathbf{p}^2}{2m} + V(r)$ and show that

$$\left\langle \frac{p^2}{m} \right\rangle = \langle \mathbf{r}\cdot\nabla V(r)\rangle.$$

Use the above result to calculate

$$\left\langle \frac{1}{r} \right\rangle_{n,\ell} \quad \text{and} \quad \langle T\rangle_{n,\ell} = \left\langle \frac{p^2}{2m} \right\rangle_{n,\ell},$$

where the subscripts refer to an average for a given $n$ and $\ell$. Use this last result to show that for the Coulomb potential $\langle T\rangle = -\frac{1}{2}\langle V\rangle$.