PY 451: Quantum Physics I   Problem Set 7

Due date: Friday, March 6, 2009, by 5:00pm

1. (text 6-5 & 6-6) For the simple harmonic oscillator, calculate $\langle m|x|n\rangle$ and $\langle m|p|n\rangle$ for arbitrary integers m and n, where x and p are the position and momentum operators. Here the bra ($\langle\cdots|$) ket ($|\cdots\rangle$) notation is a useful shorthand for quantum mechanics that we'll be using during the rest of the semester. For example

$$\langle m|x|n\rangle = \int_{-\infty}^{\infty} u_m^*(x)\, x\, u_n(x)\, dx\,,$$

   where $u_n(x)$ is the n^{th} energy eigenfunction of the simple harmonic oscillator. The definition of $\langle m|p|n\rangle$ is similar, except that the momentum operator is sandwiched between wavefunctions.

2. (text 6-10) For the simple harmonic oscillator, calculate $\langle n|x^2|n\rangle$ and $\langle n|p^2|n\rangle$. Give a physical explanation for the meaning of these two results. Specifically discuss the dependence of your results on n. Do you recover the classical limit as $n \to \infty$?

3. (text 6-15) Use Eq. (6-58) in the text to calculate the eigenfunctions of the simple harmonic oscillator, $u_n(x)$, for $n = 1$, 2, and 3. Be sure to respect the ordering of the operators x and $\frac{d}{dx}$ in the expansion of $(a^\dagger)^k$.