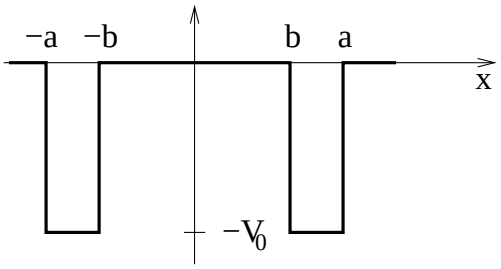
PY 451: Quantum Physics I Problem Set 5

Due date: Friday, February 20, 2009, by 4:00pm

1. (adapted from text 4-11 & 13 and Griffiths 2.47) Consider a finite double square-well potential shown in the figure.



   (a) Show that the eigenvalue conditions may be written as:

$$\tan q(a-b) = \frac{q\kappa(1+\tanh\kappa b)}{q^2-\kappa^2\tanh\kappa b}$$
$$\tan q(a-b) = \frac{q\kappa(1+\coth\kappa b)}{q^2-\kappa^2\coth\kappa b}$$

   for even and odd solutions respectively. Here $E=-\hbar^2\kappa^2/2m$, $E+V_0=\hbar^2q^2/2m$.

   (b) Show that the eigenvalue conditions approach those of the single well as $b\to 0$.

   Suppose that $|V_0|$ and $a-b$ are sufficiently large that several bound states would exist for a single well of depth $-V_0$ and width $w=a-b$. For these remaining sections, use qualitative reasoning. No calculations allowed!

   (c) Sketch the ground state wave function and the first excited state wave function for the cases: (i) $b=0$, (ii) $b\gg w$, and (iii) $b\approx w$.

   (d) Sketch the ground state and first excited state energies, E_1 and E_2 as a function of b for $b\in[0,\infty]$.

   (e) The double well is a primitive model for an electron in a diatomic molecule (the wells represent the attraction of the electron by the nuclei). If the nuclei are free to move, they will adopt the minimum energy configuration. Using the result of the previous section, does the electron tend to draw the nuclei together or push them apart? Ignore the effect of the repulsion between the two nuclei.