PY 451: Quantum Physics I   Problem Set 4

Due date: Friday, February 13, 2009, by 4:00pm

1. (Griffiths 2.39) Consider an infinite square-well potential in $[0, a]$.
   (a) Show that an arbitrary wave function $\Psi(x,t)$ of a particle in this potential returns to its initial form, $\Psi(x,t) = \Psi(x,0)$ after a quantum **revival time** $T = 4ma^2/\pi\hbar$.
   (b) Compute the *classical* revival time for a particle of energy E that is bouncing back and forth between reflecting walls at 0 and a.
   (c) For what value of E are the two revival times equal?

2. (Griffiths 2.25) Check that the wave function for the bound state of the delta-function potential well is consistent with the Heisenberg uncertainty relation. That is, compute the variances of the position and momentum, $\langle(\Delta x)^2\rangle$ and $\langle(\Delta x)^2\rangle$ and show that $\Delta x\,\Delta p \approx \hbar$. *Partial answer* (that you should compute): $\langle p^2\rangle = (mV_0/\hbar)^2$.

3. (Griffiths 2.27) Consider the double delta-function potential

$$V(x) = -V_0[\delta(x+a) + \delta(x-a)] \qquad \text{with } V_0 > 0.$$

   (a) Sketch the potential.
   (b) Determine the number of bound states for this potential.
   (c) Find the allowed energies for $V_0 = \hbar^2/ma$ and for $V_0 = \hbar^2/4ma$, and sketch the wave functions for these two cases

4. (adapted from Griffiths 2.35) A particle of mass m and energy $E > 0$ travels to the right in free space and approaches an abrupt potential drop $-V_0$ at $x = 0$.
   (a) Compute the probability that the particle is reflected back into the region $x < 0$.
   (b) When a free neutron enters a nucleus, it experiences a sudden drop in potential energy from $V = 0$ to $V \approx -12$ MeV. Suppose that a neutron of energy 4 MeV, produced by a fission event, hits a nucleus with a potential of -12 MeV. Using the one-dimensional model of part (a), what is the probability that the neutron will be absorbed by the nucleus, thereby initiating another fission event?

5. (Griffiths 2.40) A particle of mass m is in the potential well

$$V(x) = \begin{cases} \infty & x < 0 \\ -32\hbar^2/ma^2 & 0 \le x \le a \\ 0 & x > a \end{cases}$$

   (a) How many bound states exist?
   (b) In the highest-energy bound state, calculate the probability P that the particle would be found *outside* the well ($x > a$). *Answer:* $P \approx 0.542$.