PY 451: Quantum Physics I Problem Set 3

Due date: Friday, February 6, 2009, by 4:00pm

1. (Text 3-5) Consider an electron in an infinite box of dimension 10Å.

   (a) What is the energy difference (in eV) between the ground and first excited states?

   (b) Suppose that the transition from the $n = 2$ to the $n = 1$ state is accompanied by the emission of a photon, as given by the Bohr rule. What is the wavelength of the emitted photon?

2. (Text 3-9) A particle is localized in the left half of the interval $[-a/2, a/2]$. Inside the interval the potential is zero and outside the potential is infinite. The particle wave function is

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{a}} & -\frac{a}{2} < x < 0 \\ 0 & 0 < x < \frac{a}{2} \end{cases}$$

   (a) Will the particle remain localized within the left half of the interval at later times? Explain.

   (b) Calculate the probabilities that an energy measurement yields: (i) the ground-state energy and (ii) the energy of the first excited state.

3. (Text 3-12) A particle is in the ground state of an infinite square-well potential with walls at $x = \pm a$. Very suddenly, the walls are moved to $x = \pm b$ (with $b > a$). Compute the probabilities that the particle will be in: (i) the ground state of the new potential, and (ii) the first excited state of the new potential. (Hint: the last part does not require any calculation.)

4. (Text 3-14) The wave function of a particle in free space is initially

$$\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2}$$

   (a) What is the probability that the particle momentum is in the range $(p, p + dp)$?

   (b) Compute the expectation value of the energy. Give a rough argument to estimate this energy that is based on Heisenberg uncertainty principle.

5. Using the Heisenberg uncertainty principle, estimate how long a pencil can be balanced on its tip before it falls over. *Note: This is challenging problem that is meant for extra credit.*