PY 451: Quantum Physics I Problem Set 2

Due date: Friday, January 30, 2009, by 4:00pm

1. (Text 2-1) Given that $A(k) = N/(k^2 + \alpha^2)$, calculate $\psi(x)$. Plot $A(k)$ and $\psi(x)$ and show that $\Delta k \Delta x > 1$, independent of the choice of α.

2. (Text 2-5) Consider the spreading of a Gaussian wave packet for a free particle, with $\omega = \hbar k^2/2m$. Calculate the fractional change in the size of the wave packet in one second if:

   (a) The packet represents an electron with rest energy $mc^2 \approx 0.5\,\mathrm{MeV}$, with the wave packet having dimensions of: (i) $10^{-6}\,\mathrm{m}$, and also (ii) $10^{-10}\,\mathrm{m}$.

   (b) The packet represents an object of mass 1 gm and size 1 cm. [It will be convenient to express the width in units of $\hbar/mc$, where m is the mass of the particle represented by the packet.

3. (Text 2-8) Calculate the momentum-space wave function $\phi(p)$ for the wave function $\psi(x) = Ae^{-\mu|x|}$.

4. (Text 2-12 & 13) Consider the following distribution of grades in a class of 60 students:

Grades	60	55	50	45	40	35	30	25	20	15	10	5
# students	1	2	7	9	16	13	3	6	2	0	1	0

   (a) Plot a histogram of this distribution

   (b) Calculate the average grade $\langle g \rangle$ in the class.

   (c) Calculate the variance $\sigma^2 \equiv \langle g^2 \rangle - \langle g \rangle^2$.

   Now compare your histogram with a distribution of the form

$$N(g) = C\, e^{-(g-\langle g \rangle)^2/\sigma^2} \ ,$$

   with C chosen so that $\int_0^\infty N(g)\, dg = 60$.

5. (Text 2-16 & 17) Calculate $\langle x^n \rangle$ for $n = 1, 2$ for the wave function

$$\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2/2} \ .$$

   Can you quickly write down the result for $\langle x^{17} \rangle$? Next, calculate the Fourier transform $\phi(p)$ of the wave function $\psi(x)$. Calculate $\langle p^n \rangle$ for $n = 1, 2$.