PY 451: Quantum Physics I   Problem Set 11

Due date: Tuesday, April 21 2009, in class

1. (Griffiths 4.25) If the electron were a classical solid sphere of radius equal to the classical electron radius, $r_c = e^2/mc^2$, how fast would a point on the "equator" of the electron be moving? Does a model of a solid, spinning electron of radius r_c make sense? Explain. (*Hint:* Although Griffiths suggests writing the speed in m/sec, it is more convenient to express the speed in units of the speed of light c.)

2. (Griffiths 4.28) For the most general normalized spinor $\chi = \binom{a}{b}$, with $|a|^2 + |b|^2 = 1$, compute $\langle S_i \rangle$ and $\langle S_i^2 \rangle$, for $i = x, y, z$. Check that $\langle S_x^2 \rangle + \langle S_y^2 \rangle + \langle S_z^2 \rangle = \langle S^2 \rangle$.

3. (text 10-6) Consider a spin-$\frac{1}{2}$ particle whose spin wave function is represented by the normalized spinor $\frac{1}{\sqrt{65}}\binom{4}{7}$. What is the probability that a measurement of S_y yields the value $-\hbar/2$?

4. (text 10-7) Use the basic properties of the Pauli matrices to prove that

$$(\sigma \cdot \mathbf{A})(\sigma \cdot \mathbf{B}) = \mathbf{A} \cdot \mathbf{B} + i\sigma \cdot (\mathbf{A} \times \mathbf{B})$$

for any 2×2 matrices $\mathbf{A}$ and $\mathbf{B}$. Here $\sigma = (\sigma_x, \sigma_y, \sigma_z)$.

5. (text 10-8) A spin-$\frac{1}{2}$ particle is in an eigenstate of S_x, with eigenvalue $\hbar/2$ at time $t = 0$. At this time, the particle is placed in a magnetic field of magnitude B that points in the z-direction. The particle is allowed to precess in this field for a time T. At time T, the direction of the magnetic field is suddenly changed so that it points in the y-direction. After another time interval T has elapsed, a measurement of S_x is performed. What is the probability that the value $\hbar/2$ will be found?