PY 451: Quantum Physics I Problem Set 10

Due date: Tuesday, April 14 2009, in class

1. (Griffiths 4.13)

   (a) Calculate $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron that is in the ground state of hydrogen. Express your results in terms of the Bohr radius.

   (b) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the the ground state of hydrogen. (*Hint:* Use symmetry of the ground state to relate $\langle x^2 \rangle$ to $\langle r^2 \rangle$.)

   (c) Calculate $\langle x^2 \rangle$ for an electron in the state $(n, \ell, m) = (2, 1, 1)$ (*Warning:* This state is not spherically symmetric. Use $x = r \sin\theta \cos\phi$.)

2. (Griffiths 4.14) What is the *most probable* value of r for an electron in the ground state of hydrogen? (*Hint:* First figure out the probability that the electron will be found between r and $r + dr$.)

3. (Griffiths 4.17) Consider the earth-sun system as a gravitational analog of the hydrogen atom.

   (a) What is the potential energy function that corresponds to $V(r) = -e^2/r$? Assume that the mass of the earth and sun are m and M, respectively.

   (b) Determine the analog of the Bohr radius a_g for this system. Compute its numerical value.

   (c) Write the gravitational analog of the Bohr formula. By equating E_n to the classical energy of the earth in a circular orbit of radius r_0, show that $n = \sqrt{r_0/a_g}$. From this result estimate the quantum number n of the earth.

4. (text 8-11) The electron in the hydrogen atom is prepared in a state in which the wave function is

$$\Psi(\mathbf{r}) = \left(\frac{\beta}{\sqrt{\pi}}\right)^{3/2} e^{-\beta^2 r^2/2} \ .$$

   Write an expression for the probability that the electron will be found in the ground state of the hydrogen atom. More ambitiously and for extra credit, estimate this probability in the limits of $\beta \to 0$ and $\beta \to \infty$.