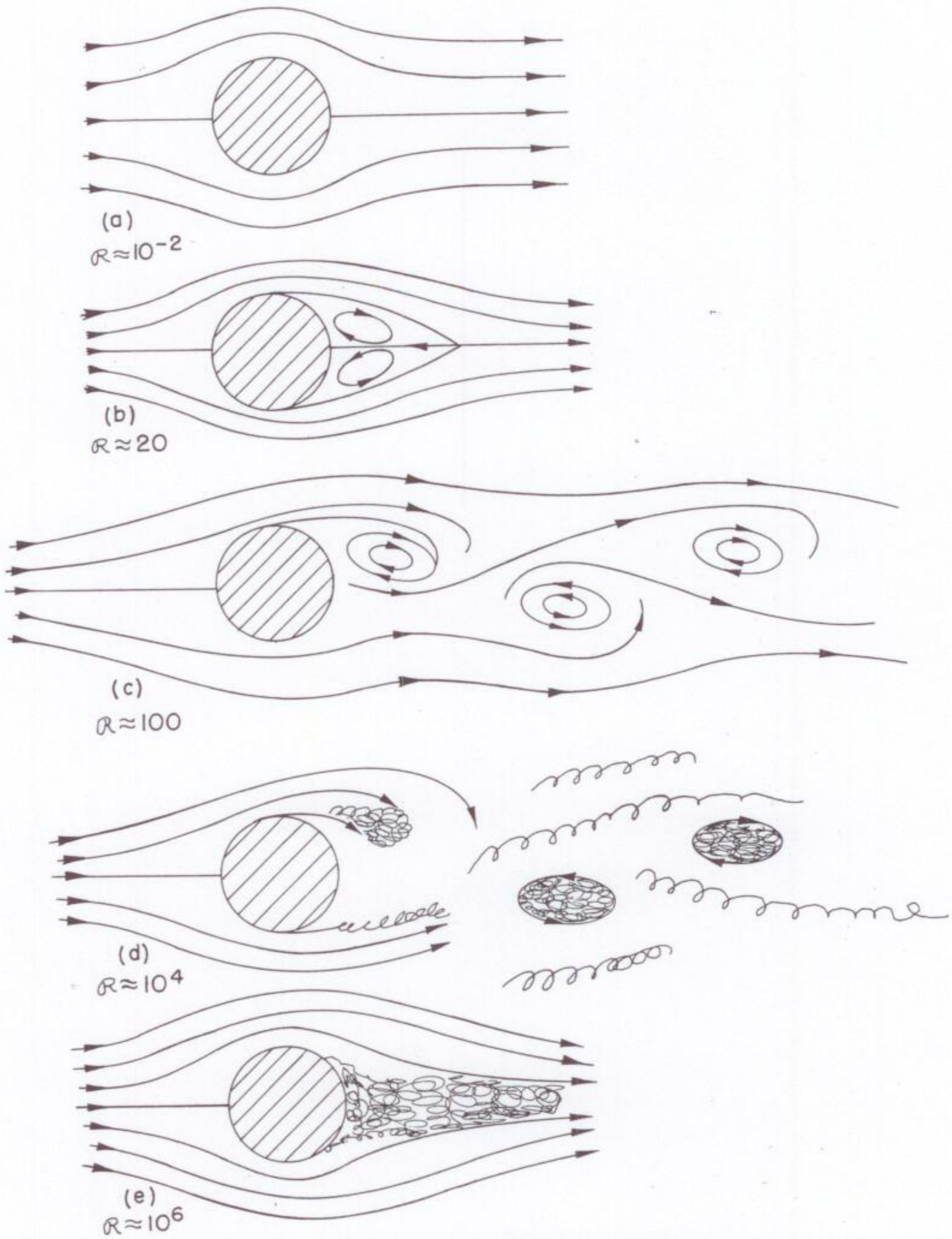

Fig. 41–6. Flow past a cylinder for various Reynolds numbers.

a circulation there even at the smallest Reynolds number or whether things suddenly change at a certain Reynolds number. It used to be thought that the circulation grew continuously. But it is now thought that it appears suddenly, and it is certain that the circulation increases with $\mathcal{R}$. In any case, there is a different character to the flow for $\mathcal{R}$ in the region from about 10 to 30. There is a pair of vortices behind the cylinder.

The flow changes again by the time we get to a number of 40 or so. There is suddenly a complete change in the character of the motion. What happens is that one of the vortices behind the cylinder gets so long that it breaks off and travels downstream with the fluid. Then the fluid curls around behind the cylinder and makes a new vortex. The vortices peel off alternately on each side, so an instantaneous view of the flow looks roughly as sketched in Fig. 41–6(c). The stream of

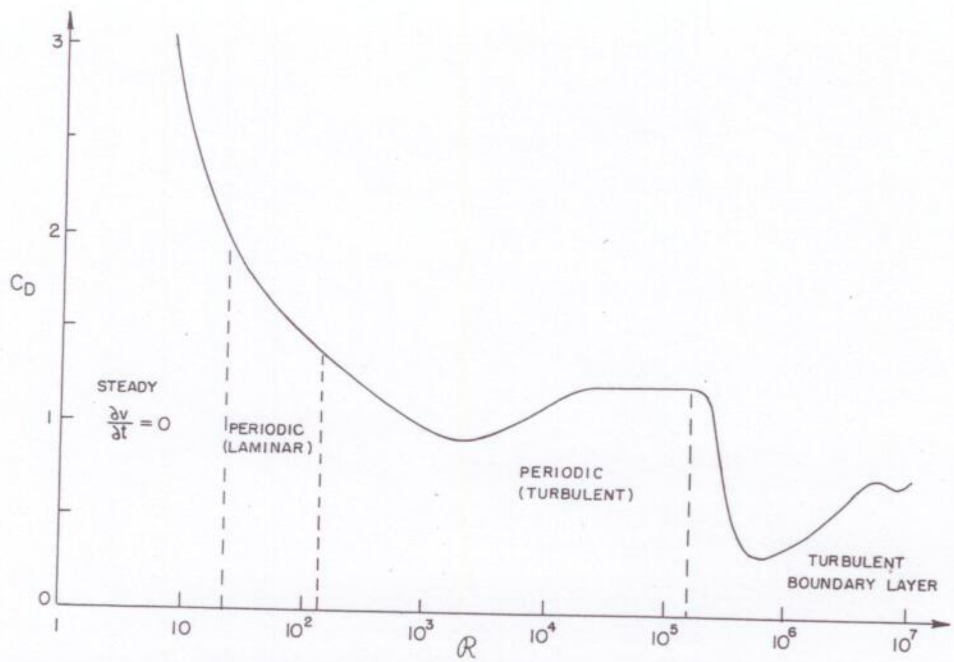

Fig. 41–4. The drag coefficient C_D of a circular cylinder as a function of the Reynolds number.

41–4 Flow past a circular cylinder

Let's go back to the problem of low-speed (nearly incompressible) flow over the cylinder. We will give a qualitative description of the flow of a real fluid. There are many things we might want to know about such a flow—for instance, what is the drag force on the cylinder? The drag force on a cylinder is plotted in Fig. 41–4 as a function of $\mathcal{R}$—which is proportional to the air speed V if everything else is held fixed. What is actually plotted is the so-called *drag coefficient* C_D, which is a dimensionless number equal to the force divided by $\frac{1}{2}\rho V^2 Dl$, where D is the diameter, l is the length of the cylinder, and ρ is the density of the liquid:

$$C_D = \frac{F}{\frac{1}{2}\rho V^2 Dl}.$$

The coefficient of drag varies in a rather complicated way, giving us a pre-hint that something rather interesting and complicated is happening in the flow. We will now describe the nature of flow for the different ranges of the Reynolds number. First, when the Reynolds number is very small, the flow is quite steady; that is, the velocity is constant at any place, and the flow goes around the cylinder. The actual distribution of the flow lines is, however, not like it is in potential flow. They are solutions of a somewhat different equation. When the velocity is very low or, what is equivalent, when the viscosity is very high so the stuff is like honey, then the inertial terms are negligible and the flow is described by the equation

$$\nabla^2\boldsymbol{\Omega} = 0.$$

This equation was first solved by Stokes. He also solved the same problem for a sphere. If you have a small sphere moving under such conditions of low Reynolds number, the force needed to drag it is equal to $6\pi\eta aV$, where a is the radius of the sphere and V is its velocity. This is a very useful formula because it tells the speed at which tiny grains of dirt (or other particles which can be approximated as spheres) move through a fluid under a given force—as, for instance, in a centrifuge, or in sedimentation, or diffusion. In the low Reynolds number region—for $\mathcal{R}$ less than 1—the lines of $\boldsymbol{v}$ around a *cylinder* are as drawn in Fig. 41–5.

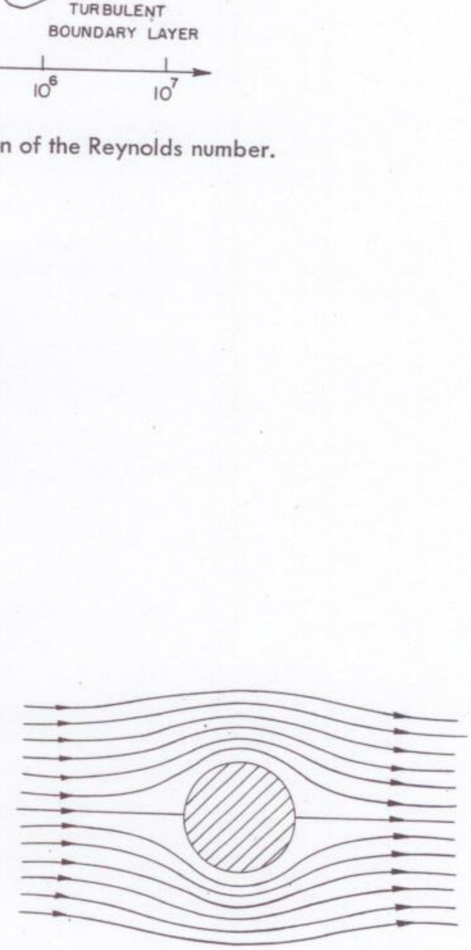

Fig. 41–5. Viscous flow (low velocities) around a circular cylinder.

If we now increase the fluid speed to get a Reynolds number somewhat greater than 1, we find that the flow is different. There is a circulation behind the sphere, as shown in Fig. 41–6(b). It is still an open question as to whether there is always