PY 251 - Principles of Physics

Mathematical preliminaries.

Evaluate the expressions that follow, covering the right hand side of the equations. If you miss more then 10%-15% of the answers, take a note of the expressions you cannot evaluate correctly and please come see me.

Derivatives and integrals.

$$\frac{d}{dt}\Big(\frac{at^2}{2} + bt + c\Big) = at + b \quad (1)$$

$$\frac{d}{dt}\sin(\omega t + \phi_0) = \omega\cos(\omega t + \phi_0) \quad (2)$$

$$\frac{d}{dx}\log(5x) = \frac{1}{x} \quad (3)$$

$$\int 3x\,dx = \frac{3x^2}{2} + c \quad (4)$$

$$\int_{t_0}^{t_1}(gt + v)\,dt = \frac{g}{2}(t_1^2 - t_0^2) + v(t_1 - t_0) \quad (5)$$

$$\int_0^t A\sin(\omega t')\,dt' = -\frac{A}{\omega}[\cos(\omega t) - 1] \quad (6)$$

$$\int_{t_1}^{t_2}\frac{dx}{dt}\,dt = x(t_2) - x(t_1) \quad (7)$$

Vectors.

Let

$$\begin{aligned} \vec{v}_1 &= 3\hat{\imath} + \hat{\jmath} - 2\hat{k} \\ \vec{v}_2 &= -\hat{\imath} + 2\hat{\jmath} \\ \vec{v}_3 &= 2\hat{\jmath} + \hat{k} \\ \vec{w} &= -2\hat{\imath} - \hat{\jmath} + \hat{k} \end{aligned}$$

then

$$\vec{v}_1 + \vec{v}_2 = 2\hat{\imath} + 3\hat{\jmath} - 2\hat{k} \tag{8}$$

$$2\vec{v}_3 - \vec{v}_2 = \hat{\imath} + 2\hat{\jmath} + 2\hat{k} \tag{9}$$

$$\vec{v}_1 + \vec{v}_2 + \vec{v}_3 - \vec{w} = 4\hat{\imath} + 6\hat{\jmath} - 2\hat{k} \tag{10}$$

$$\vec{v}_1 \cdot \vec{v}_2 = -1 \tag{11}$$

$$\vec{v}_2 \cdot \vec{w} = 0 \tag{12}$$

$$w \equiv \sqrt{\vec{w} \cdot \vec{w}} = \sqrt{6} \tag{13}$$

$$\vec{v}_2 \times \vec{v}_3 = 2\hat{\imath} + \hat{\jmath} - 2\hat{k} \tag{14}$$

Vectors in component notation.

Let

$$\begin{aligned} \vec{u} &= \begin{pmatrix} a \\ 2b \\ -a \end{pmatrix} \\ \vec{v} &= \begin{pmatrix} b \\ c \\ 2a \end{pmatrix} \end{aligned}$$

then

$$2\vec{u} - \vec{v} = \begin{pmatrix} 2a - b \\ 4b - c \\ -4a \end{pmatrix} \tag{15}$$

$$\vec{u} \cdot \vec{v} = ab + 2bc - 2a^2 \tag{16}$$

$$\vec{u} \times \vec{v} = \begin{pmatrix} 4ab + ac \\ -2a^2 - ab \\ ac - 2b^2 \end{pmatrix} \tag{17}$$

Time dependent vectors.

Let

$$\begin{aligned} \vec{v} &= a\hat{\imath} - gt\hat{k} \\ \vec{r} &= A\cos(\omega t)\hat{\imath} + B\sin(\omega t)\hat{\jmath} \end{aligned}$$

then

$$\frac{d\vec{v}}{dt} = -g\hat{k} \tag{18}$$

$$\frac{d\vec{r}}{dt} = -A\omega\sin(\omega t)\hat{\imath} + B\omega\cos(\omega t)\hat{\jmath} \tag{19}$$

$$\int_{t_0}^{t_1} \vec{v}\,dt = a(t_1 - t_0)\hat{\imath} + \frac{g}{2}(t_1^2 - t_0^2)\hat{k} \tag{20}$$

$$\int \vec{r}\,dt = \frac{A}{\omega}\sin(\omega t)\hat{\imath} - \frac{B}{\omega}\cos(\omega t)\hat{\jmath} + \vec{c} \tag{21}$$