PY 251 - Principles of Physics

Lecture notes - September 6, 2005.

Suggestions for learning the course material.

- Attend all lectures. Pay attention. Taking notes is not as important as following the explanations. I will post on the web (http://physics.bu.edu/∼rebbi) schematic notes, like this one, with the important concepts and formulae.

- Ask questions, in class, whenever something is not clear.

- Study the notes as soon as possible after the lecture, while memory of the lecture is still fresh. Study, do not just read: you should be able to put the notes aside and reproduce all that is contained in them.

- After going through the material presented in class, read the appropriate pages in the textbook.

- Start your homework only after having completed the above steps.

- Take advantage of office hours.

- For the homework, use intuition, the mathematical formalism and good common sense.

Position, velocity and acceleration.

We identify the position of a point-like object by an origin O and a displacement vector $\vec{r}$ from O to the object.

To make the definition of $\vec{r}$ quantitative we choose three mutually orthogonal unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ and express $\vec{r}$ in terms of its components along the unit vectors

$$\vec{r} = r_1 \hat{\imath} + r_2 \hat{\jmath} + r_3 \hat{k} \tag{1}$$

If we introduce a set of Cartesian axes with origin in O and directions along $\hat{\imath}, \hat{\jmath}, \hat{k}$, then r_1, r_2, r_3 are also the Cartesian coordinates of the location of our object, but it is better to think of vectors as abstract objects without reference to a Cartesian coordinate system.

Alternative notations are possible

$$\begin{aligned}\vec{r} &= r_1\hat{i}_1 + r_2\hat{i}_2 + r_3\hat{i}_3 \\ \vec{r} &= r_x\hat{x} + r_y\hat{y} + r_z\hat{z} \\ \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k}\end{aligned}$$

etc. Note that the last notation $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ would normally only be used with a displacement vector.

When we assign values to the components, rigorously we should include units, e. g.

$$\vec{r} = 1.2\text{m}\,\hat{i} + 3\text{m}\,\hat{j} - 2.5\text{m}\,\hat{k} \tag{2}$$

since the displacement vector carries dimensions of length. When there is no possibility of confusion, however, we may omit the unit of length and write simply $\vec{r} = 1.2\hat{i} + 3\hat{j} - 2.5\hat{k}$.

When an object moves, the vector $\vec{r}$ varies with time. Thus $\vec{r}$ and its components are functions of time t: $\vec{r} \equiv \vec{r}(t), \quad r_i \equiv r_i(t)\ i = 1, 2, 3$.

If an object is at $\vec{r}_1$ at time t_1 and at $\vec{r}_2$ at time t_2, then its displacement during the time interval $\Delta t = t_2 - t_1$ is $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$. The ratio

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \tag{3}$$

is the average velocity of the object during the interval Δt. In the limit $\Delta t \to 0$ this becomes the instantaneous velocity, or simply the velocity of the object

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} \tag{4}$$

In term of components

$$v_i(t) = \frac{dr_i(t)}{dt}, \quad i = 1, 2, 3 \tag{5}$$

We can proceed with the velocity is the same way as we have done with the displacement vector and consider the variation of the velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ during a finite time interval $\Delta t = t_2 - t_1$. We can then define an average acceleration, and an instantaneous acceleration or simply acceleration

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} \tag{6}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} \tag{7}$$

with

$$a_i(t) = \frac{dv_i(t)}{dt}, \quad i = 1, 2, 3 \tag{8}$$

One normally does not proceed further (by considering variations of acceleration etc.) because, as we will see, the acceleration of an object depends on the forces acting on it, which in turn depend on its position and possibly velocity. Thus with the acceleration the system closes:

$\vec{r}$ and $\vec{v}$ determine $\vec{a}$ which in turn determines the variation of $\vec{v}$ (intuitively speaking the value of $\vec{v}$ an infinitesimal amount of time dt later), and $\vec{v}$ in turn determines the variation of $\vec{r}$ (again figuratively speaking the value of $\vec{r}$ at $t + dt$), etc.

Motion in one dimension.

We consider the motion of a point-like object that moves along a straight line. We can take this to be a line through O oriented along $\hat{k}$ and thus we have

$$\vec{r}(t) = z(t)\hat{k} \tag{9}$$

Accordingly, we have

$$\vec{v}(t) = v(t)\hat{k} \tag{10}$$

$$\vec{a}(t) = a(t)\hat{k} \tag{11}$$

where, to simplify the notation, we have written $v(t), a(t)$ for $v_3(t), a_3(t)$.

From Eqs. 4,7 we get

$$v(t) = \frac{dz(t)}{dt} \tag{12}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2z(t)}{dt^2} \tag{13}$$

We consider now a few typical motions.

1) Motion with uniform acceleration:

$$z(t) = -\frac{g}{2}t^2 + v_0 t \tag{14}$$

Equations 12,13 give

$$v(t) = -gt + v_0 \tag{15}$$

$$a(t) = -g \tag{16}$$

and we see that this is a motion with constant acceleration equal to $-g$. Figure 1 illustrates such a motion with $g = -10\text{m/s}^2$, a close approximation to the acceleration of gravity, and $v_0 = 20\text{m/s}$. In this figure we have introduced

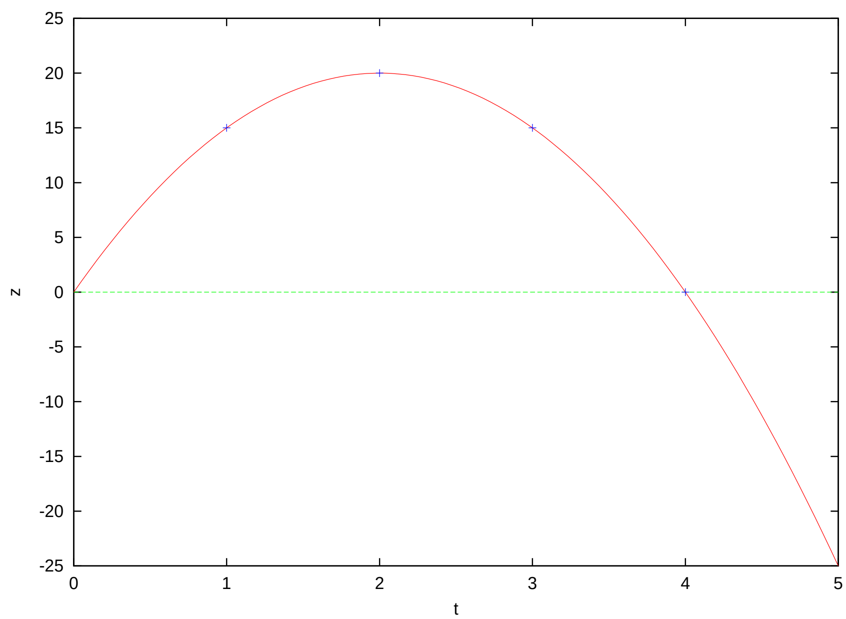

Figure 1: Motion with uniform acceleration.

a time axis, thus each point of the graph represents a point in space-time. In the language of relativity one often refers to points in space-time as events (thinking of something that happens at that point in space and that moment in time). Thus the space-time trajectory in Fig. 1 is a continuous collection of space-time events. The dots in the figure show the events with $t, x = (1, 15), (2, 20), (3, 15), (4, 0)$, with x in meters and t is seconds.

We can easily find the maximum height reached by the object thrown in the air with initial velocity x_0. The maximum is reached when the derivative with respect to t of $z(t)$ vanishes, which is the same as $v(t) = 0$ (intuitively obvious). Equation 15 gives $t_{max} = v_0/g$ and the corresponding value of z is $z_{max} = v_0^2/(2g)$.

2) Harmonic motion:

$$z(t) = A\sin(\omega t) \tag{17}$$

Equations 12,13 give

$$v(t) = A\omega\cos(\omega t) \tag{18}$$

$$a(t) = -A\omega^2\sin(\omega t) \tag{19}$$

In this motion the object oscillates around the point $z = 0$ and the space-time trajectory is a sinusoid. The period T is given by the condition $\omega T = 2\pi$, thus $T = 2\pi/\omega$. An important feature of this motion is that the acceleration is proportional in magnitude to the displacement, with proportionality factor ω^2, and has opposite orientation.

3) Motion of a body falling through a medium which offers a resistance proportional to the velocity (linear drag).

Forces and their effects on the motion will be considered later. Here we just assume that the body has an acceleration given by a constant negative term $-g$ and a further term with magnitude proportional to the velocity and opposite direction: thus $a(t) = -g - kv(t)$. We want to solve for the motion. We have

$$\frac{dv(t)}{dt} = a(t) = -g - kv(t) \tag{20}$$

or

$$\frac{dv}{-g - kv} = dt \tag{21}$$

Integrating we get

$$\int_{v_0}^{v_1} \frac{dv}{-g - kv} = \int_{t_0}^{t_1} dt \tag{22}$$

where we denoted by t_0 and t_1 the initial and final values of time and by v_0, v_1 the corresponding values of v. The integrals can be done exactly and we get

$$\Big[-\frac{1}{k}\log(\mathrm{abs}(-g - kv))\Big]_{v_0}^{v_1} = \Big[-\frac{1}{k}\log(g + kv)\Big]_{v_0}^{v_1} = t_1 - t_0 \tag{23}$$

For simplicity let us take $t_0 = 0$ and $v_0 = 0$. Equation 23 thus becomes

$$-\frac{1}{k}[\log(g + kv_1) - \log(g)] = -\frac{1}{k}\log(1 + kv_1/g) = t \tag{24}$$

or

$$v(t) = -\frac{g}{k}(1 - e^{-kt}) \tag{25}$$

where we have written t and v for t_1 and v_1 since there is no more possibility of confusion. (Check that in Eq. 25 dimensions are correct!)

Equation 25 tells us that the motion begins as an accelerated motion with acceleration $-g$ [for $t \ll 1/k$ we can expand the exponential and get $v(t) = -g(1 - 1 + kt + \ldots)/k = -gt$]. However, as time goes on, the motion approaches exponentially a fall with constant velocity (terminal velocity) $v_\infty = -g/k$.

Proceeding in a manner analogous to what we did to find $v(t)$ we can integrate the equation for $z(t)$

$$\frac{dz(t)}{dt} = v(t) = -\frac{g}{k}(1 - e^{-kt}) \tag{26}$$

to obtain

$$z_1 - z_0 = -\frac{g}{k}\int_{t_0}^{t_1} (1 - e^{-kt})\, dt = -\frac{g}{k}\Big[t + \frac{1}{k}e^{-kt}\Big]_{t_0}^{t_1} \tag{27}$$

Setting again $t_0 = 0, t_1 = t, z_1 = z(t)$ the equation above gives

$$z(t) = z_0 - \frac{gt}{k} - \frac{g}{k^2}(e^{-kt} - 1) \tag{28}$$

Reading.

Read Fishbane Chapter 2 pages 28-52.