The first excited state should be doubly-degenerate

- **Lanczos only gives one state out of a degenerate multiplet**

Go back to the Krylov space

\[ H^m |\Psi\rangle = \sum_k C_k E_k^m |\Psi_k\rangle \]

If states \( k, j \) are degenerate, we have a term

\[ E_j^m (C_j |\Psi_j\rangle + C_k |\Psi_k\rangle) \]

For any \( m \), this vector points in the same direction in the subspace spanned by \( |\Psi_j\rangle, |\Psi_k\rangle \)

Acting with \( H \) cannot “separate” degenerate states

Since the Lanczos basis spans the same Krylov space, we only get one state out of a degenerate multiplet of states

- the particular linear combination depends on the initial state

Numerical round-off errors can lead to apparent degeneracies (multiple copies of the same state). This indicates that the scheme breaks down as the basis becomes non-orthogonal.
Following last Friday’s discussion, we went through the program “random” in detail. Please read the commented code and explore what it’s doing.