Motion in two or three dimensions

Vector equations of motion

$$\dot{\vec{x}}(t) = \vec{v}(t)$$

$$\dot{\vec{v}}(t) = \frac{1}{m}\vec{F}[\vec{x}(t), \vec{v}(t), t]$$

Each component forms its own equation; all coupled through **F**

Example: Planetary motion (2D plane)

$$\vec{F}(r) = -\frac{GMm}{r^3}\vec{r}$$

We should use reduced mass; one-body problem for $\mu = \frac{Mm}{(m+M)}$

- but if M >>>> m we can use m

$$\dot{x} = v_x$$
$$\dot{v}_x = -GMx/r^3$$
$$\dot{v}_y = -GMy/r^3$$
$$\dot{y} = v_y$$

$$r = \sqrt{x^2 + y^2}$$

Applied in hw-3 (assignment reviewed in class)

Leapfrog procedure:

$$x(n+1) = x(n) + \Delta_t v_x(n+1/2)$$
$$y(n+1) = y(n) + \Delta_t v_y(n+1/2)$$
$$v_x(n+1/2) = v_x(n-1/2) - \Delta_t GMx(n)[x^2(n) + y^2(n)]^{-3/2}$$
$$v_y(n+1/2) = v_y(n-1/2) - \Delta_t GMy(n)[x^2(n) + y^2(n)]^{-3/2}$$