The “let” block; a simple way to store values in functions

We often want to store the “internal state” of some function
without having to pass that state as an argument

For example, rand() can be called without any argument
- but clearly there must be some internal state that is somehow saved

References to data (pointers) can be permanently saved in “let” blocks
- functions defined inside a let block can access these pointers

Example, part of letblock.jl (random number generator, inside a module)

```
let
    r = Ref(convert(UInt64,1))
    global function ran64()
       r[]=r[]*a+c
    end
end
```

r is a reference (pointer) to an unsigned integer
- the value at r is accessed by r[]
- would be r[i] for element i of a 1-dim array

Why not just use r declared in the global scope?
- for efficiency, avoid using global variables

The function must be declared global to make it accessible outside let-end
- global function objects are treated as constants, not slowing things down
- the integers a and c are declared as constants before let

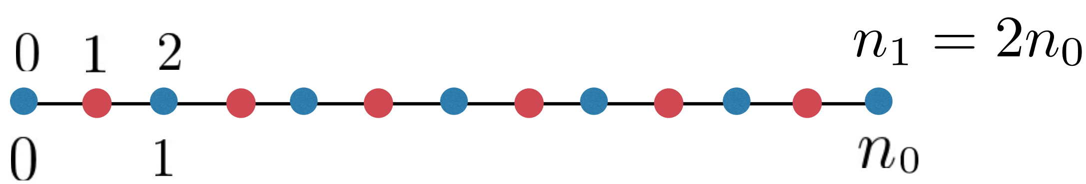
The let block is a local hard scope, many other uses (see Julia doc)

Romberg integration

Idea: Use two or more trapezoidal integral estimates, extrapolate

- step sizes (decreasing order) $h_0, h_1, \ldots, h_m$, integral estimates $I_0, I_1, \ldots, I_m$
- use polynomial of order n to fit and extrapolate to h=0
- error for given h scales as h^2 (+ higher even powers only)
- use polynomial P(x) with $x=h^2$

Simplest case: 2 points (m=1), using $h_0=(b-a)/n_0$ and $h_1=h_0/2$ ($x_1=x_0/4$)



Function evaluation once only for each point needed

$$I_0 = I_\infty + \epsilon x_0, \quad I_1 = I_\infty + \epsilon x_0/4$$

$$\rightarrow \quad I_\infty = \frac{4}{3} I_1 - \frac{1}{3} I_0 \quad + O(h_0^4) \quad [O(x_0^2)]$$

reducing h by 50%

- error should be 1/4 of previous
- ε is unknown factor, eliminated

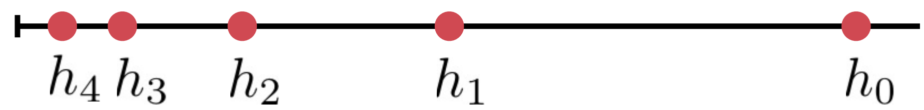
Computation cost doubled, error reduced by two powers of h_0!

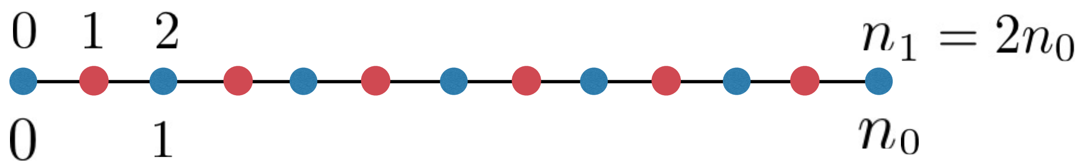
Generalizes easily to the case of m estimates (Friday)

General case; $h_0, h_1, \ldots, hm \rightarrow I_1, I_2, \ldots, I_m$

For each i, let $h_{i+1} = h_i/2$ ($x_{i+1} = x_i/4$)

- save old sum, add new points





How to construct a polynomial of order n going throug n+1 point pairs (x_i,y_i)

$$P(x) = \sum_{i=0}^{m} y_i \prod_{k \neq i} \frac{x - x_k}{x_i - x_k}$$

$x_i = h_i^2$

Evaluate (this is the extrapolation) at x=0 (h=0)

$$I_\infty = \sum_{i=0}^{m} I_i \prod_{k \neq i} \frac{-h_0^2 2^{-2k}}{h_0^2 (2^{-2i} - 2^{-2k})} = (-1)^m \sum_{i=0}^{m} I_i \prod_{k \neq i} \frac{1}{2^{2(k-i)} - 1}$$

Error decreases very rapidly: $O(h^{2(m+1)})$

Implemented in romberg.jl (both closed and open cases)