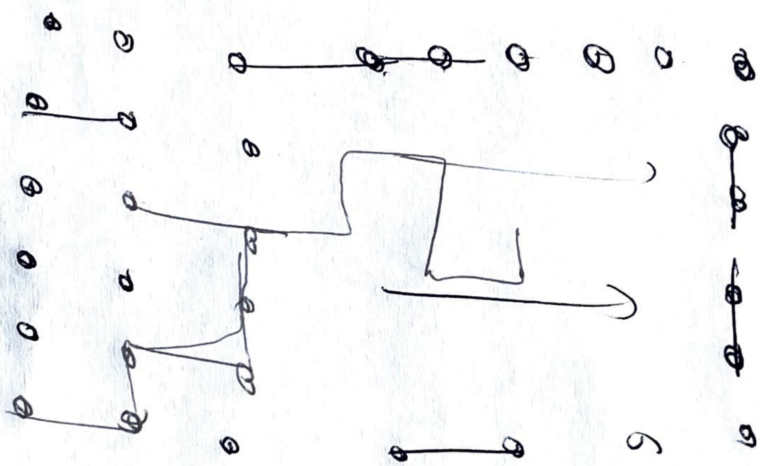
Lecture 4 Sol-Gel Transition

$\boxed{\text{Crowick } 36}$

We will now use similar techniques to SAW to think about "percolation transitions" and in particular the Sol-gel transition...

A brief overview of percolation... Consider a graph

"Bond Percolation"



Throw bonds randomly with probability b

There is an interesting transition.. called percolation

Imagine increasing b ~~....~~

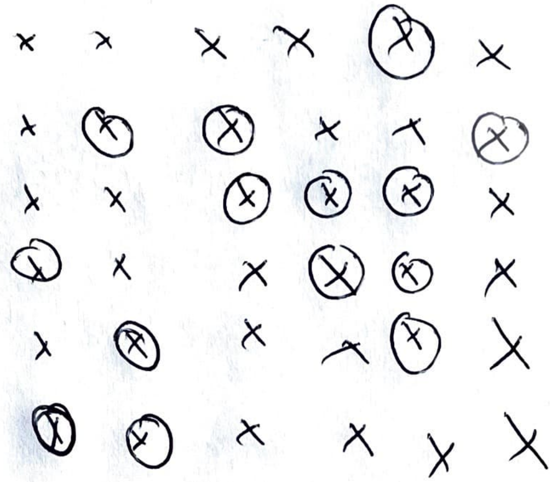
You start getting clusters of sites that are connected.....

At some $b = b_c$, a giant (infinite) cluster emerges that spans whole system. This is bond percolation transition

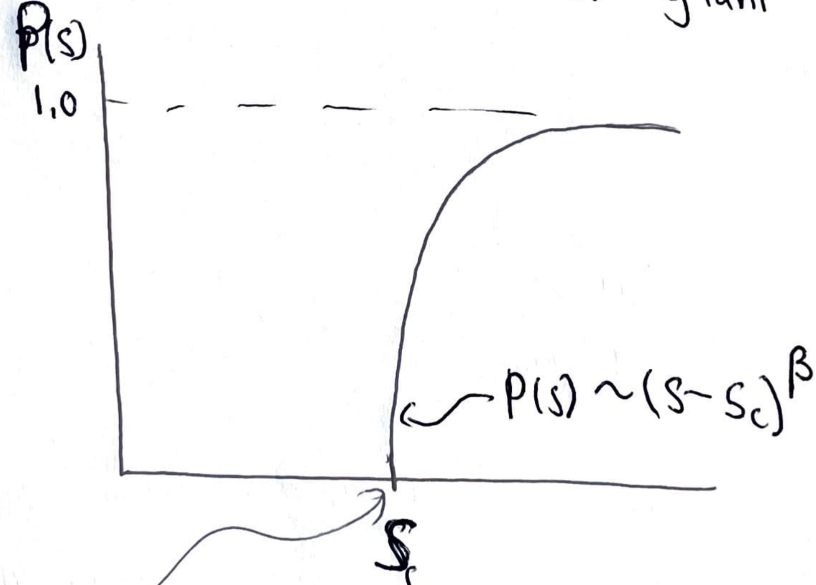
This called the bond percolation transition.

Can also do "site percolation"

A site is occupied with probability "s"



Define order parameter P: probability that site is part of giant cluster

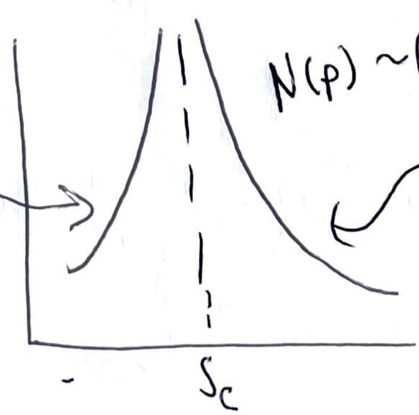


"Site perocaltion transition"

$N(p) \sim (s - s_c)^{-\gamma}$

Probability to belongs to finite cluster

Belongs to large finite cluster that doesn't quite span

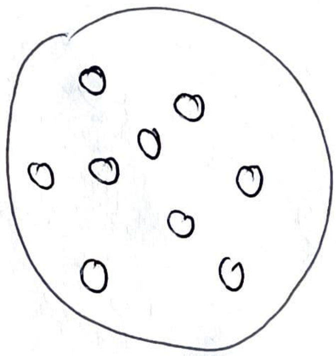


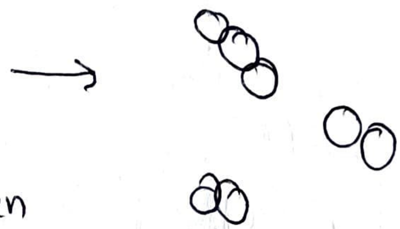
everything is in large cluster

One of the most physically interesting example Is the Sol-gel transition...

There are two kinds of molecules...

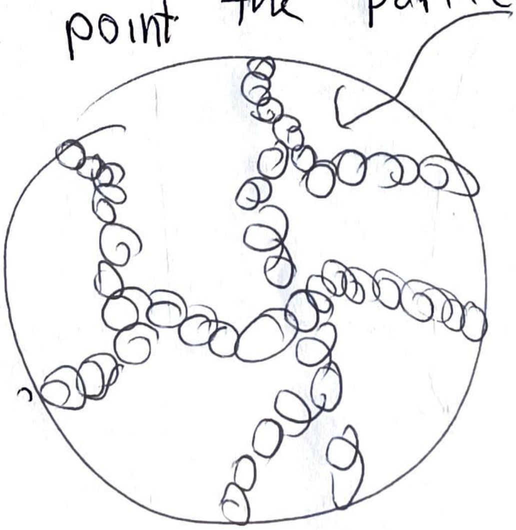
"solvent" and "particles"
gel





When
particles
get near each
other they can bond
to each other

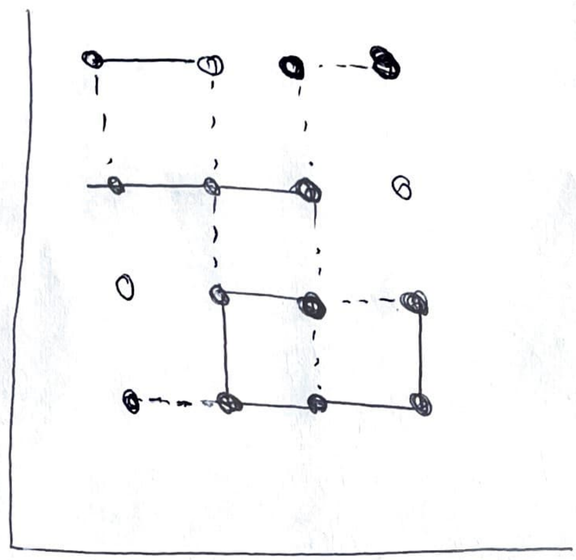
At some point the particles span the system



System goes from
~~being~~
behaving like
a "liquid" (solution)
Sol
and a viscoelastic
solid

Lets do RG and construct a phase diagram of the sol-gel transition..

A simple model for this is a lattice



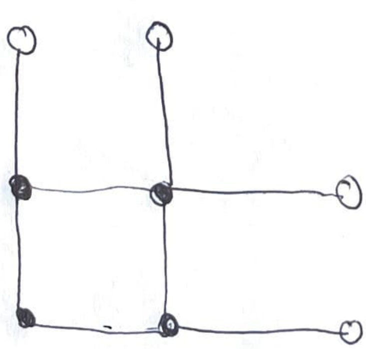
Each site can be occupied by gel molecule with probability S

At sites $\rightarrow$ $\begin{cases} S & \text{gel molecule} \\ 1-S & \text{solvent} \end{cases}$

At bonds $\begin{cases} \text{If two sites are both occuppied have bond with prob } b \end{cases}$

<u>Approximation</u>: Everything is independent..

We will do RG where we coarse grain and the rescale... What is our scheme



$b=2$

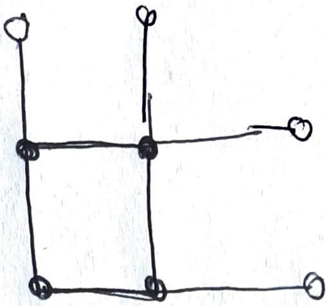
"symmetric" Rule

Site is occuppied site if a <u>connected cluster</u> spans from <u>left to right</u> or <u>top or bottom</u>

So lets calculate

4 sites occuppied

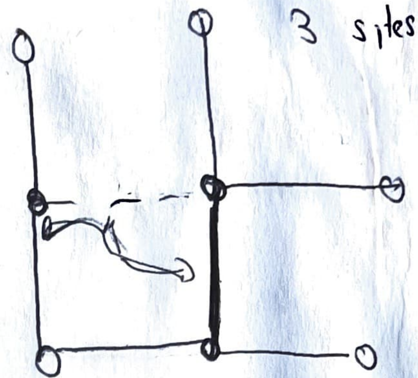
$S' =$



$S^4[1-(1-b)^4]$

no bonds span

3 sites occuppied

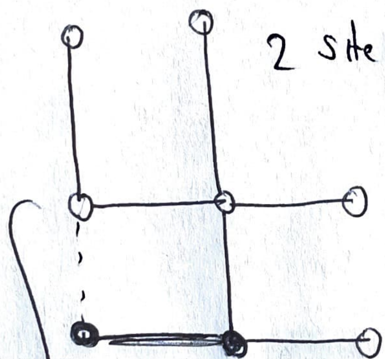


$4S^3(1-S)[1-(1-b)^2]$

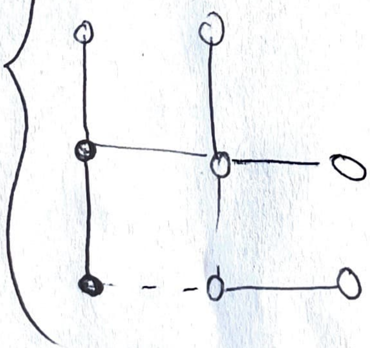
4 missing

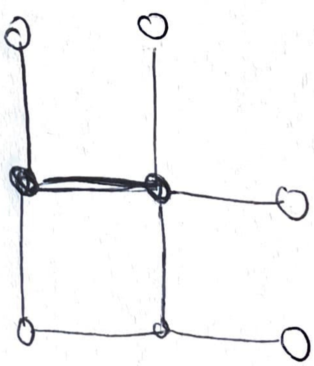
Both possible bonds are absent

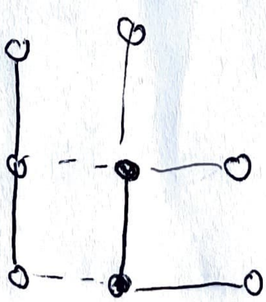
2 site occuppied



$4S^2(1-S^2)b$







$$S' = S^4[1-(1-b)^4] + 4S^3(1-S)[1-(1-b)^2] + 4S^2(1-S)^2 b$$

Lets' renormalize bond

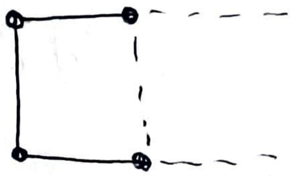
4 sites

at least one of those

$(1-(1-b)^2)$

(Either terminal) bond

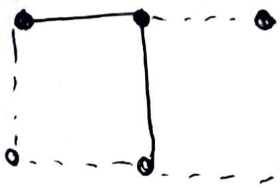
$s^4 b^4$



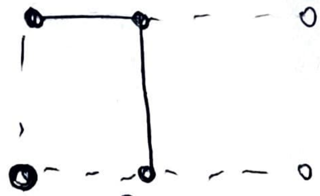
$4 s^4 b^3 (1-b)$

$s^4 b^2 (1-b)^2$

$s^4 b^2 (1-b)^2$



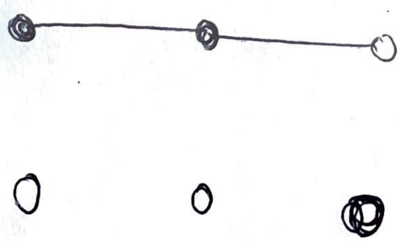
$s^4 b^2 (1-b)^2$



$\cancel{4}\, 2 s^3 (1-\cancel{b}) b^2$

3 sites

Particular terminal bond



$\boxed{2 s^2 (1-s^2) b^2}$

$2 b^2 (1-b) s^3 (1-s)$

$2 s^3 (1-s) b^2$

This gives

$$s'b' = \overset{(1-(1-b)^2)}{\cancel{\;\;}}\; s^4\left[b^4 + 4b^3(1-b) + 3b^2(1-b)^2 + 2s^3(1-s)b^2\right]$$

$$+\, 2s^4\left[b^3(1-b)^2 + b^2(1-b)^3\right]$$

$$+\, 2s^3(1-s)_4 b^2(2-b) + 2s^2(1-s^2)b^2$$

Rewriting other RG equation..

$$s' = s^4\left[1-(1-b)^4\right] + 4s(1-s)\left[1-(1-b)^2\right] + 4s^2(1-s)^2 b$$

Notice $s=1 \;\; b=1$ is fixed point (Gel)

$s=0 \;\; b=0$ if fixed point (Solution)

$s^* = 0.879$ $\quad (s_c^*, b_c^*)$ ← non-trivial fixed point

$b^* = 0.586$

Linearize around this point

$$\begin{bmatrix} \delta s' \\ \delta b' \end{bmatrix} = \begin{bmatrix} \dfrac{\partial s'}{\partial s} & \dfrac{\partial s'}{\partial b} \\ \dfrac{\partial b'}{\partial s} & \dfrac{\partial b'}{\partial b} \end{bmatrix}\Bigg|_{\substack{s=s^* \\ b=b^*}} \begin{bmatrix} \delta s \\ \delta b \end{bmatrix}$$

Two eigenvalues

Relevant → $\lambda_1 = 1.60$

Irrelevant $\lambda_2 = 0.536$

(S^*, b^*) has one relevant and irrelevant direction

By looking at eigenvectors show locally looks like