Lecture 1: RG From Space...

In the first part of this class, we will focus on the "Renormalization Group" or R.G. as it is often affectionately shortened to....

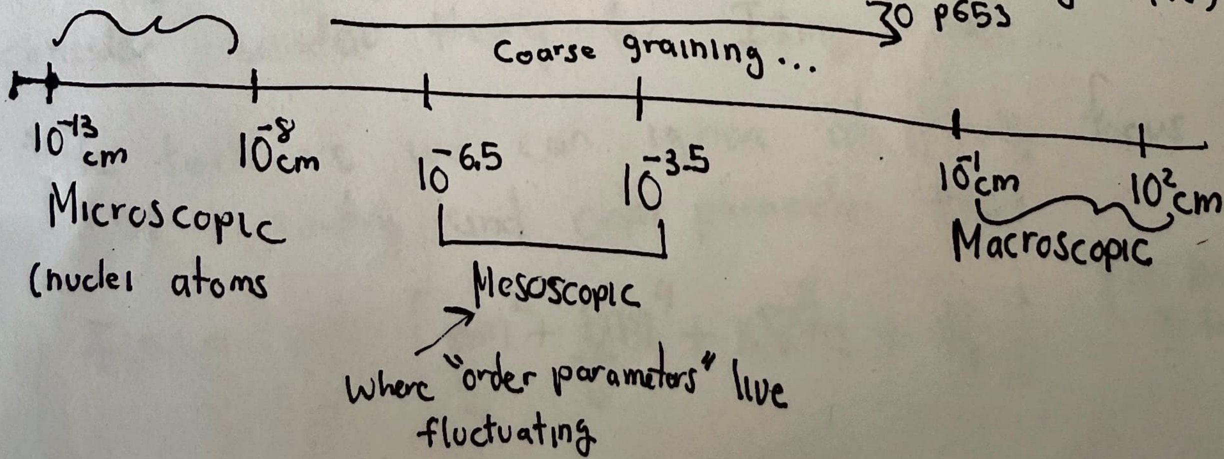
Rather than a single thing, "R.G." should be thought of as an inter-related set of ideas and techniques developed in the 1960s and 1970s to put physics – and in particular Statistical Physics – on a firm footing (Landau, Wilson, Kadanoff, M Fisher, Wegner,

At a very high level, R.G. (especially "Wilsonian RG") formalizes the idea of "effective theories"

- ~~Best~~ Earliest example of this is Landau's work

  $\Rightarrow$ We can ignore microscopic details of a system, and work with "order parameters"

Scales according to Michael Fisher (Rev. Mod Phys 1998) 70 p653



R.G. provides a quantitative way of thinking about why and how we can coarse grain....

R.G. From Space

The basic idea from space, (we will of course successively through next 8 weeks zoom in more and more) is that we can imagine changing the (length) scale on which we want to understand the properties of the system.... by for example, rescaling the system

$$\vec{x} \to b \cdot \vec{x}$$

Under such a transformation, we expect all the "short-range" stuff to "average out" and so we should be able to ignore all this "short-distance" or U.V. physics (high mometum, by uncertainty relation high energy) stuff

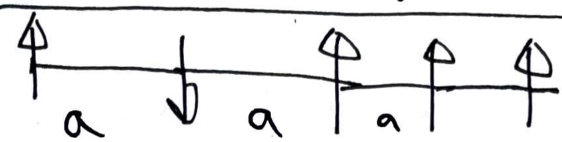
Indeed, this is what we can do... For example, consider Landau theory for Ising model...

: He told us we can ignore all details, focus of symmetry and order parameter $m(r)$

$$F(m) = \int d^d r \left[\frac{a m^2}{2} + \frac{b m^4}{4} + \kappa \nabla^2 m + F_0 \right]$$ (no knowledge of lattice)

We can now ask what happens if we change our "ruler" $\Rightarrow$ (rescaling by "b)

We expect our physics to be invariant and not depend on very small scales... like ~~say~~ the lattice spacing "a".



Length scales a (lattice)

Ruler ~ Correlation length ξ

However, it turns out that often the naive scaling assupption does not work... Somehow even though it seems like the "microscopic physics" should not matter, it still effects "long-distance physics"

R.G. offers a systematic way to properly treat this short distance Physics in a coarse-grained description

In fact R.G. says that this "short-distance" physics will "renormalize" the terms that appear in the corresponding "effective theories"

$\Rightarrow$ Moreover, we cannot talk about a single effective theory, but instead a family of effective theories

Given a scale

$$b_a^{\ell} \longrightarrow F^{\ell}$$

"effective theory at that scale"

This is really quite surprising I would say and making this statement something that allows for calculations and makes non-trivial predictions...

A central role in R.G. has been played by the study of phase-transitions and critical phenomena...

The reason is that this is the setting where the "short-distance" physics has the most dramatic effect even at long distances... (though often it has no effect at large enough scales in for example $d > 4$)

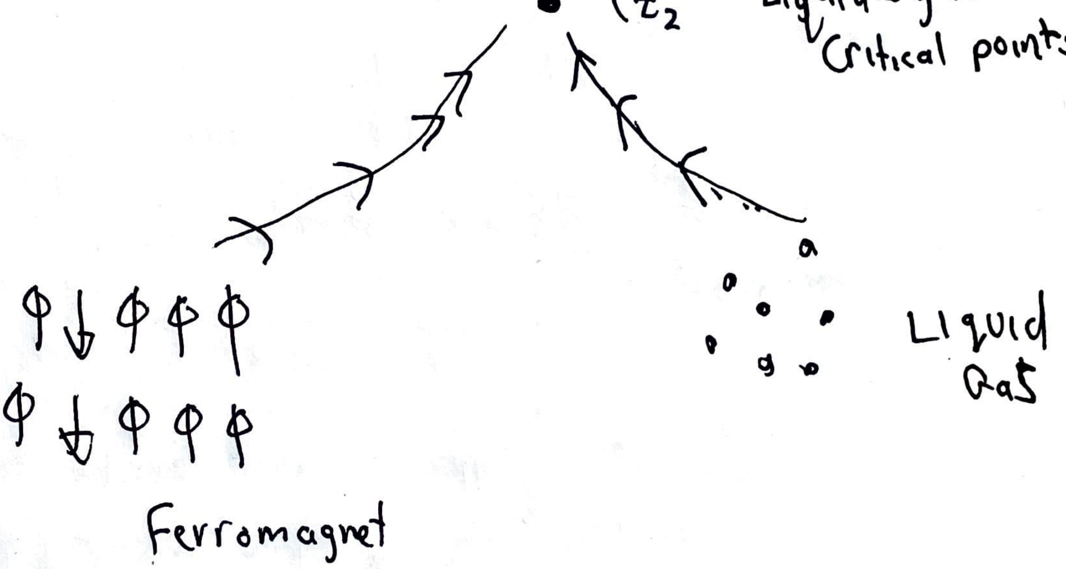
4 Different meanings of RG

Before discussing further, we are now in a position to clarify one thing that often confuses students + practioners alike...

R.G. actually is a lot of different ideas all called the same thing. What they share is this basic idea that one can construct a coarse-graining scheme to successfully treat short-distance physics

(A) A theory of (critical) fixed points and non-linear
a linear behavior near fixed points

$\Rightarrow$ An explanation for universality... Coarse grain two very different "microscopic theories"/systems converge to the same "long-distance" physics near a critical point • ($\mathbb{Z}_2$ Liquid-gas critical points)



(B) A Field-theoretic formulation of RG for critical phenomena based on Feynman diagrams and perturbation theory... (ε- expansion)

C. QFT methods originally designed to make sure physics was formulated to be "independent" of the U.V. cutoff (i.e. did not appear explicitly in any of the physical quantities)

Callan–Symanzik equations Gell-Mann–Low

D. Construction of (largely) numeric schemes to do non-perturbative R.G transformations (coarse graining steps)...

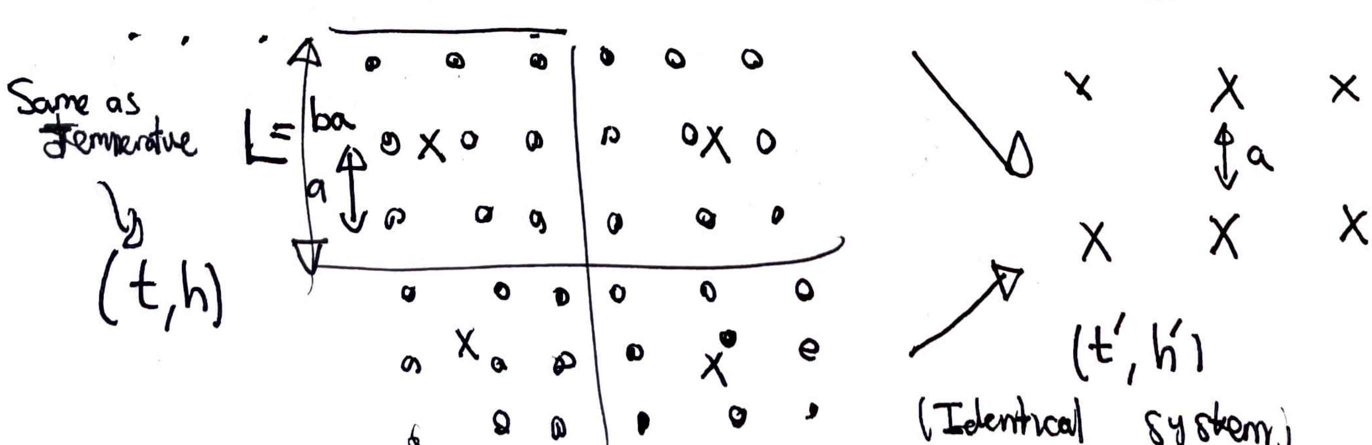
NRG, Monte-Carlo RG, Tensor-Network Renormalization..

Kadanoff and flows from space..

We will focus here primarily on Wilson's RG theory while touching of all of the above

What is Wilsonian RG. . . . $\dfrac{e^{-\beta\left(J\sum_{\langle i,i'\rangle} S_i S_{i'} - \sum_i h_i S_i\right)}}{Z}$

Consider an Ising model coarse grain... $\{x' = \frac{x}{b}\}$ (Scale



$$\bar{S}_{x'} \equiv b^{-d} \underbrace{\sum_{x \in B_{x'}} S_x}_{\text{Sum over spin in block}} = \zeta(b)\, S'_{\vec{x}'}$$

"spin rescaling" or renormalization factor

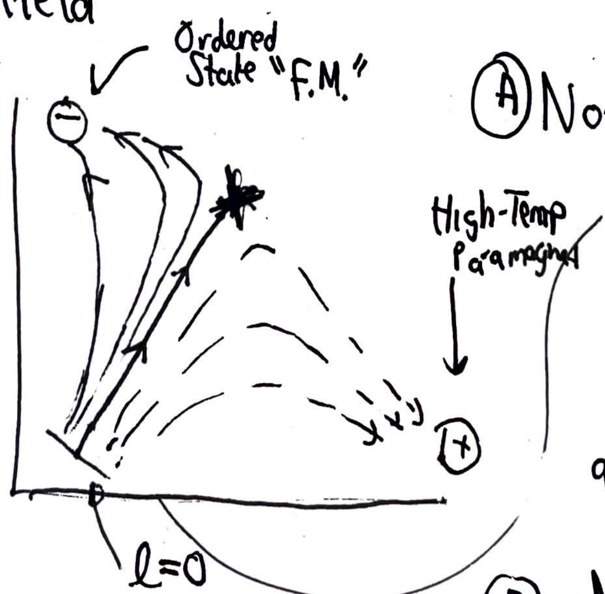
$$\Rightarrow \quad \left.\begin{aligned} t' &= \theta(b)\, t \\ h' &= \zeta(b)\, h \end{aligned}\right]$$

All that happens that temperature and field also "renormalize")

$\theta(b) \sim b^{\lambda}$

$\zeta(b) \sim b^{-\omega}$

So get "flow" in space of couplings temperature and magnetic field



Ⓐ Notice trajectories are close end up very different places as one coarse grains

Ⓑ Near fixed point have "relevant" and "irrelevant" directions...

(This picture is not quite there... See Discussion Fisher p 668-669)

Notice the special role played by the "fixed point" corresponding to the critical point. This "effective theory" Is scale invariant. It is self-similar..

For this reason a special role is played by Scale invariance and self-similarity...

This will be our starting point for making all this more concrete...