PY 895 Homework 1. **Fall 2024** **Due: Tuesday September 12th**

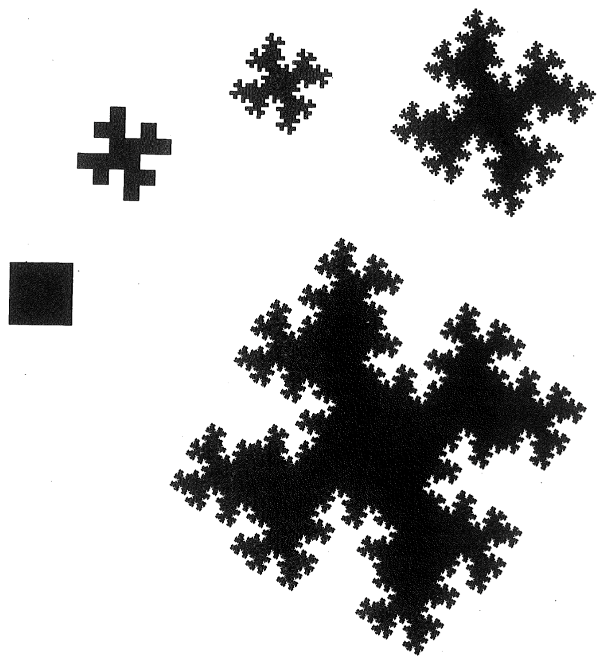

FIG. 1: Quadric Koch island for Problem 1.

1. Calculating fractal dimensions. Calculate the fractal dimension of the quadric Koch island shown in Figure 1.

2. RG for RW with Lorentzian distribution. Show that the Lorentzian distribution (in 1-dimension)

$$p_\sigma(\vec{r}) = \frac{\sigma}{\pi}\frac{1}{r^2+\sigma^2} \tag{1}$$

is a fixed point of the coarse-graining transformation that takes $\vec{r} \to \vec{r'} = \sum_{i=1}^{n} \vec{r}_i$. In other words,

$$P(\vec{r''}) = p_\sigma(\vec{r}) \tag{2}$$

for some rescaling $\vec{r''} = n^\alpha r'$.

3. Fixed point of RG transformations. Consider the transformation $R(u,v) : (u,v) \to (u',v')$ given by

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = R(u,v) = \begin{pmatrix} u - v + u^2 \\ (2-u)v \end{pmatrix}. \tag{3}$$

(a) Find the fixed points.
(b) Linearize the map about each of the fixed points and identify the number of relevant, irrelevant, and marginal directions.
(c) Sketch the phase portrait indicating the fixed points and some of the flows into and out of them. Be careful of the signs.

4.**Self-Avoiding Random Walk RG revisited.** Consider again the problem of a self-avoiding walk on the square lattice. Construct an RG scheme with zoom factor $b = 3$ (so that nine sites of the fine lattice are represented by one site of the coarse lattice). Find the RG map $K'(K)$, find its fixed points and estimate the critical exponent ν at the nontrivial fixed point K_c. Is it closer to the numerical result than the $b = 2$ scheme discussed in lecture and by Creswick? Recall, that for $b = 2$ we got $K_c = 0.379$ and $\nu = 0.715$ and numerical estimates suggest that exact answers are $K_c = 0.379$ and $\nu = 0.75$.