Statistical Physics of Disordered Systems

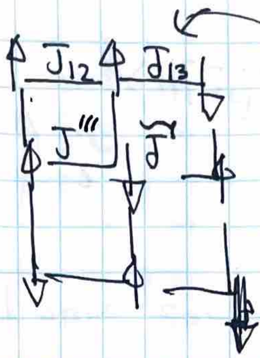
So far we have been covering systems with lots of symmetries (translation symmetry, ... ect) ... What happens when this is not the case?

Theoretical Motivations + Experimental Motivations

However, many systems are much more "complex" $\Rightarrow$

For example, outside a lattice we could imagine spin system

Motivation 1
"Disordered Couplings"



"Every coupling is different"

on some level this is much more complicated...

Instead of a single parameter $J \Rightarrow$ for a system with N spins $\Rightarrow$ need specify N^2 couplings....

Of course, we can't do this so go to a "statistical description" of J_{ij}

$$P(J_{ij}) \sim N(\bar{J}, \sigma_J^2)$$

(For example, imagine it's drawn from some Gaussian distribution with mean $\bar{J}$, std σ_J ...

when $\frac{\sigma_J}{\bar{J}} \ll 1 \Rightarrow$ return to old situation

(really $\frac{\sigma_J}{\bar{J}} \to 0$ as $N \to \infty$ since we need to think about thermodynamic limit)..

So then we are interested in physics.. for given realization $\{J_{ij}\}$

$$F(\{J_{ij}\}) = \log Z(\{J_{ij}\})$$

However, we want to think about typical behavior...

So want to average ove disorder...

Lot of potential choices:

$$\begin{cases} \langle F(\{J_{ij}\}) \rangle_{P(J_{ij})} & \text{Average free energy "quenched" average..} \\ \log \langle Z\{J_{ij}\} \rangle_{P(J_{ij})} & \text{"Average partition function".} \\ \log \mathrm{Tr}_S\, e^{-\beta \langle H[S_i; \{J_{ij}\}] \rangle_{P(J_{ij})}} & \text{Replace with "average of couplings"} \Rightarrow \text{ignore disorder..} \end{cases}$$

Really, think about subtleties but as we will see natural "self-averaging" quantity is "quenched" though hard to calculate...

$\boxed{\text{Motivation 2}}$ This idea that

Complexity = Randomness

is really idea due to Wigner and Dyson...

Towering success is Uranium atom

U has 92 protons, 146 neutrons

$$|\psi\rangle = H|\psi\rangle$$

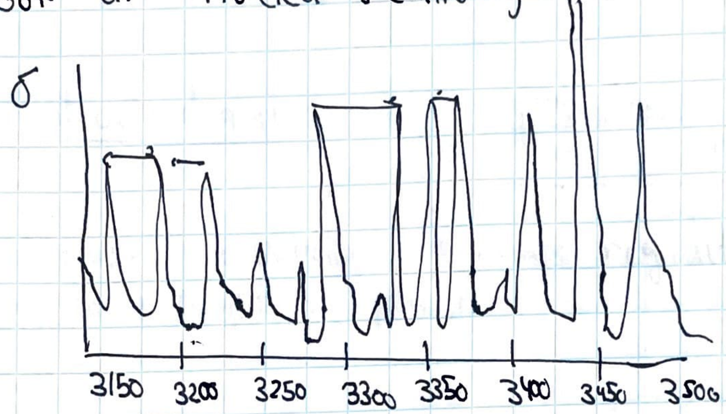
This is way too complicated..

Idea... Replace H by "Random Symmetric Matrix"

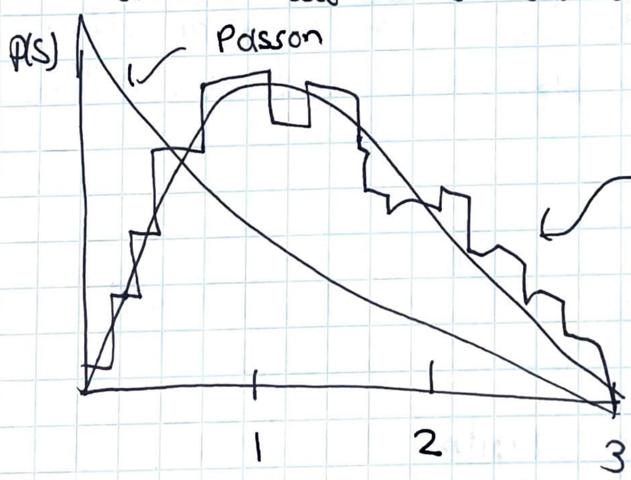
Reason, due to central limit theorem we expect that many properties become universal... don't expect stuff depend on details but general statistical properties..

Big sucess...

Look at nuclear scattering cross sections..



Look at distance between "resonances" ⇒ level spacings of Eigenvalues of H



$$P(s) = \frac{1}{2}\pi s e^{-\frac{\pi s^2}{4}}$$

Look like R.M.

Dyson ~~Wigner~~ described this as a "new kind of statistical mechanics ~~where~~ in which we renounce exact knowledge not of state of a system, but the nature of the system itself."

⇒ Inspired lots of thinking in ecology

May ⇒ Physicist turned ecologist..

we will return to this..

Exam

Motivation 3

Structural and Spin Glasses.. (Lubchenko + Wolynes Ann. Rev. Phy Chem 2007)

As usual, theoretical considerations are fine and dandy but we are interested in physical observations.. One of the most interesting...

Structural glass $\Longleftrightarrow$ amorphous substances that are rigid

Weird because we usually associate rigidity with Symmetry Breaking to a crystaline phase...

So one of the most common and interesting ways to get a disordered system that look rigid is to supercool a liquid...

What is different about amorphous structures from periodic lattice?

We expect thermal fluctuations to easily allow system to jump from one amorphous state to other so system should flow...

So what is the basic phenomenology of glass transition...

"out of equilibrium" $\Longrightarrow$ ages $\underbrace{G(t,t') \neq G(t-t')}_{\text{"Aging"}}$

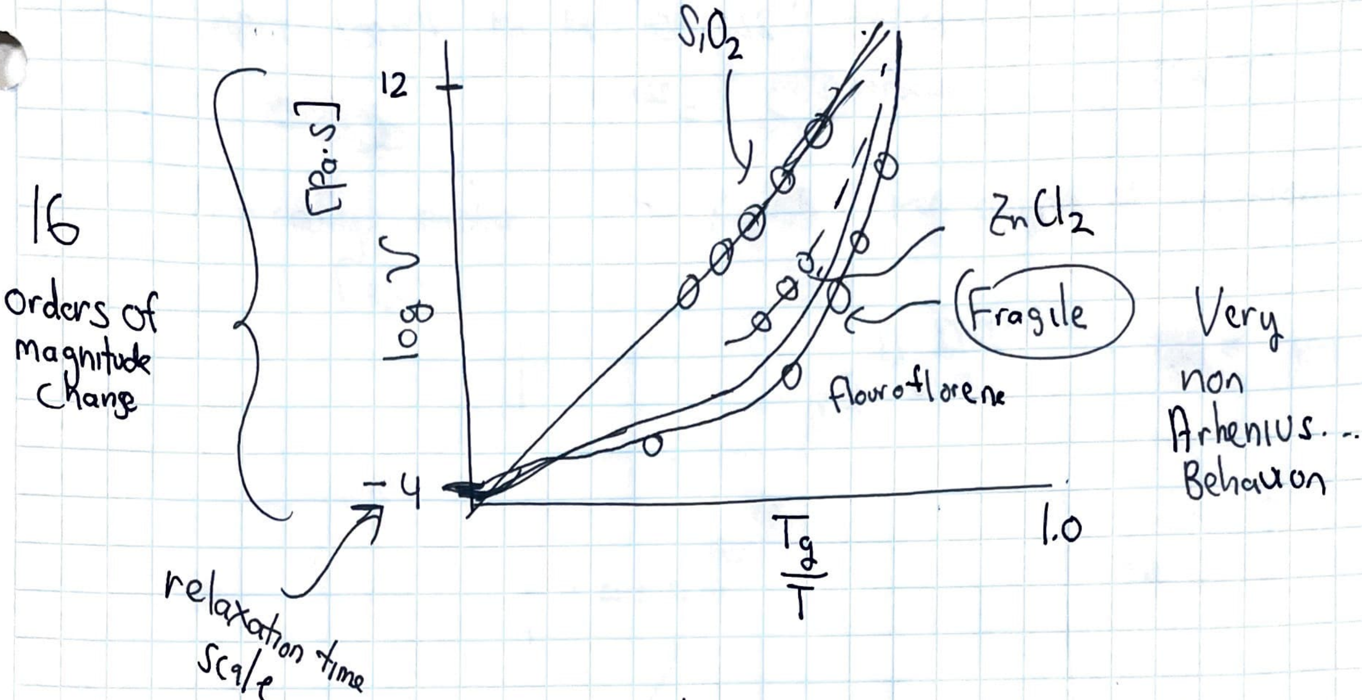
T_K T_g T_A T_v

Kauzmann Temperature

glass temperature

vaporization

Look at "Angell" plot of viscosities



If one measures ~~excitation~~ relaxation times... (lowest frequency peak of dialectrical constant)

Volger-Flucher Law

$$\tau = \tau_0 e^{\frac{DT_0}{T-T_0}}$$

D is "fragility"

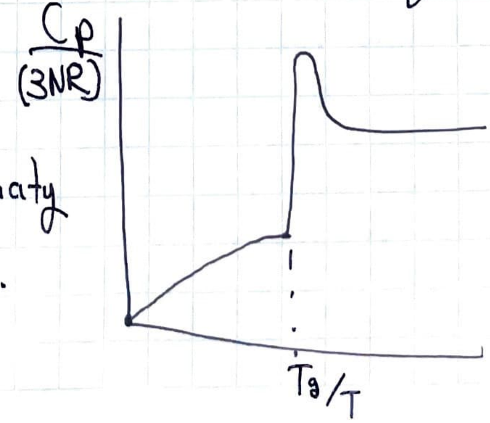
So this diverges, but not as power law...

Something fundamentally different from what we considered..

So "kinetically trapped" in configurations...

But some of the mystery is that it is also related to thermodynamics.. Consider supercooled liquid...

Look at heat capacity

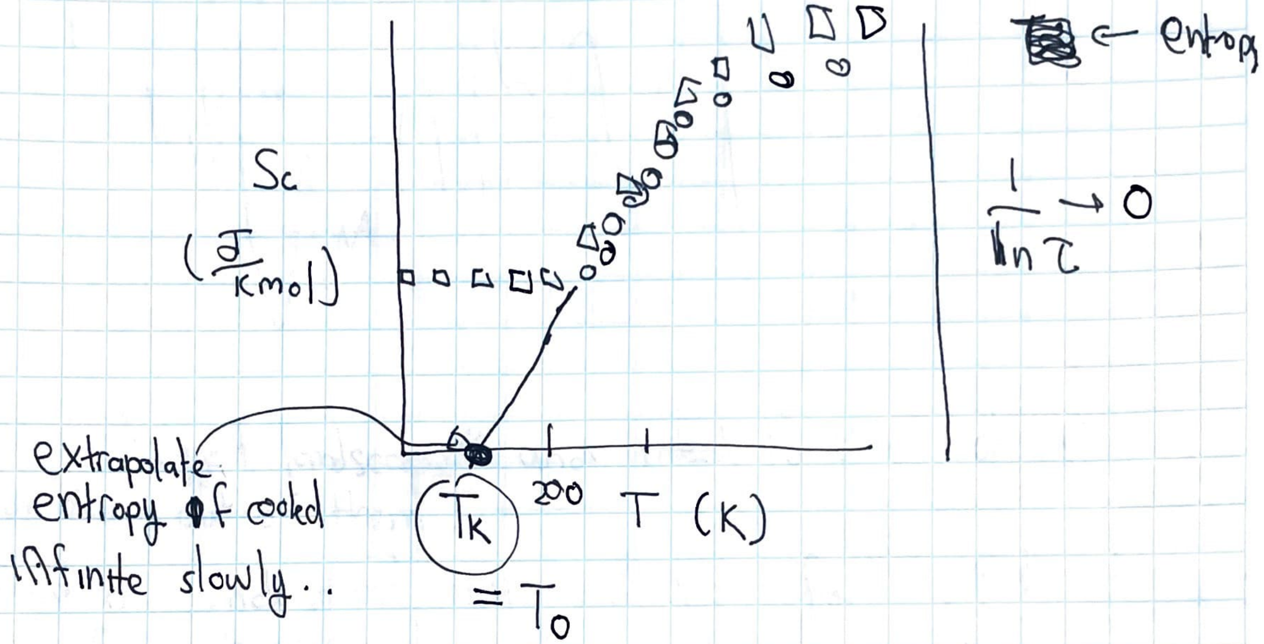


Recall integral of heat capacity is change in entropy...

There seems to be "excess" entropy $\Rightarrow$

$$\Delta S^{excess} = S^{supercooled} - S^{crystal}$$

Typically order few $\sim k_B$ per degree of freedom



So have funny situation where there seems to kinetic trapping in states with extra D.O.F.s

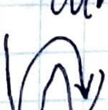
What are the D.O.Fs?

Anderson, Varma, Halperin

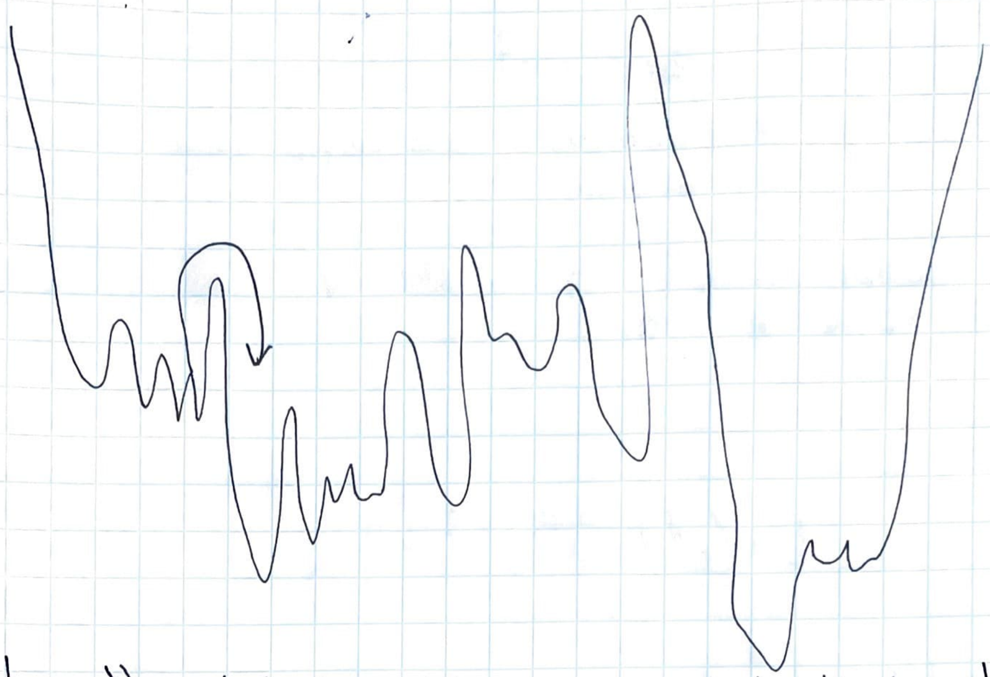
Wolynes R.F.O.T.

Trapped in local D.O.F.s that have to thermally excited..

Going to lower energy configuration requires overcoming activation barrier



Motivates looking at energy landscapes and Kinetics



Have "rugged landscape" and need to think about tunneling out of them...

Kind of understand this well now for infinite-dimension "mean-field" glasses but for real glasses $d=2,3$...

Who knows how this exactly works..

Parisi vs Fisher Huse..

Motivation 4: Computer Science, Machine Learning, Statistical Learning, Neuroscience, Biophysics...

Thinking about energy landscapes inspired lots of work in optimization, neuroscience (Hopfield Model) Machine Learning..

Basic idea.. Want to minimize some function

$$E(\underset{\sim}{x}, \theta_i)$$

dof

Random quenched variables

For example,

in Hopfield model $\Longrightarrow$ memories to be stored as minima

Random K-sat problem $\Longrightarrow$ Radom clauses that need to be satisfied..

Calculatate

$$F[\theta_i] = \log \int \mathcal{D}x \, e^{-\beta[E(x,\theta_i) + h_i x_i]}$$

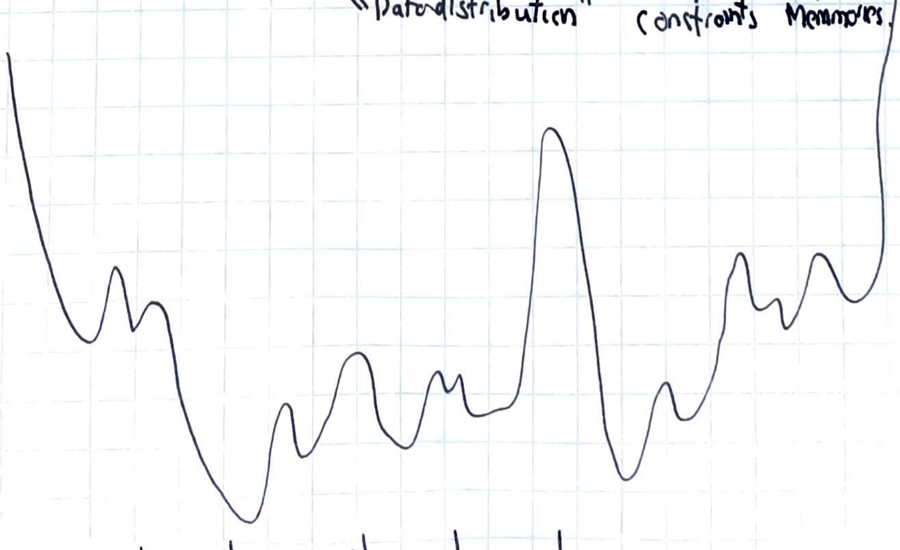
Generate moments of solutions by getting cumulant generating function...

$$\langle x_i \rangle = \frac{\partial F}{\partial h_i}$$

$$\langle\langle x_i^2 \rangle\rangle - \langle x_i \rangle^2 \approx \frac{\partial F}{\partial h_i \partial h_j}$$

Understand how landscape changes as "Disorder" changes

"Datadistribution" constraints Memories



Often get algorithmic phase transitions . . .