# *PY 541 Problem Set 7: HW Exercises Due on Nov 22*

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Read the assigned material before class. Pre-class questions are due midnight the night before before Tuesday and Thursday classes and noon on Wednesdays when homeworks are not due (usually Thursdays), and otherwise should be turned in with the homeworks.

Welcome to PY 541 Statistical Physics. This is a graduate course that assumes you have taken an undergrad course on these topics (e.g. Thermal Physics, Statistical Physics, etc.). It also assume some basic familiarity with core mathematical areas: probability, linear algebra, vector calculus as well rudimentary programming skills (Python, Mathematica, etc). If you do not feel comfortable with these concepts, please come talk to me.

All exercises are from second edition of Jim Sethna's book available at:
https://www.lassp.cornell.edu/sethna/StatMech/index.html.
There are also hints and code for computational exercises at this website.

## *Readings and Pre-class questions*

**Tuesday:**
Read Chapter 9, Sec. 9.1-9.2
Pre-class question: 9.9 Ising order parameter
In-class: 9.10 Nematic Defects, Problems 9.6

**Wednesday:**
Read: Chapter 9.3, 9.4
Pre-class question: 9.2 Classifying topological Defects
In-class: Problem 9.2 - discussion

**Thursday:**
Read: Chapter 10.1-10.5
Pre-class question: None
In-class question: Problem 10.10; Overview of chapter

**Tuesday:**
Read Chapter 10.5-10.9
Pre-class question: None
In-class: Problem 10.3

## Homework Exercises

9.5 Landau theory for the Ising model.
9.13 Chiral Wave Equation.
9.20 Number and phase in superfluids.

**1.** *The binary alloy:* A binary alloy (as in $\beta$ brass) consists of $N_A$ atoms of type $A$, and $N_B$ atoms of type $B$. The atoms form a simple cubic lattice, each interacting only with its six nearest neighbors. Assume an attractive energy of $-J$ $(J > 0)$ between like neighbors $A-A$ and $B-B$, but a repulsive energy of $+J$ for an $A-B$ pair.

(a) What is the minimum energy configuration, or the state of the system at zero temperature?

(b) Estimate the total interaction energy assuming that the atoms are randomly distributed among the $N$ sites; i.e. each site is occupied independently with probabilities $p_A = N_A/N$ and $p_B = N_B/N$.

(c) Estimate the mixing entropy of the alloy with the same approximation. Assume $N_A, N_B \gg 1$.

(d) Using the above, obtain a free energy function $F(x)$, where $x = (N_A - N_B)/N$. Expand $F(x)$ to the fourth order in $x$, and show that the requirement of convexity of $F$ breaks down below a critical temperature $T_c$. For the remainder of this problem use the expansion obtained in (d) in place of the full function $F(x)$.

(e) Sketch $F(x)$ for $T > T_c$, $T = T_c$, and $T < T_c$. For $T < T_c$ there is a range of compositions $x < |x_{\text{sp}}(T)|$ where $F(x)$ is not convex and hence the composition is locally unstable. Find $x_{\text{sp}}(T)$.

(f) The alloy globally minimizes its free energy by separating into $A$ rich and $B$ rich phases of compositions $\pm x_{\text{eq}}(T)$, where $x_{\text{eq}}(T)$ minimizes the function $F(x)$. Find $x_{\text{eq}}(T)$.

(g) In the $(T, x)$ plane sketch the phase separation boundary $\pm x_{\text{eq}}(T)$; and the so called spinodal line $\pm x_{\text{sp}}(T)$. (The spinodal line indicates onset of metastability and hysteresis effects.)

## Honor Code

All students are expected to follow the BU Honor Code. While collaboration is allowed and encouraged on HWs, each student should write up their own solutions. Copying HW is strictly forbidden. The students are allowed to consult all resources and books. However, students are NOT allowed to consult problem solutions from previous years or as found on the web.