PY 541 Problem Set 5: HW Exercises Due on Oct. 27th

Pankaj Mehta

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Read the assigned material before class. Pre-class questions are due midnight the night before before Tuesday and Thursday classes and noon on Wednesdays when homeworks are not due (usually Thursdays), and otherwise should be turned in with the homeworks.

Welcome to PY 541 Statistical Physics. This is a graduate course that assumes you have taken an undergrad course on these topics (e.g. Thermal Physics, Statistical Physics, etc.). It also assume some basic familiarity with core mathematical areas: probability, linear algebra, vector calculus as well rudimentary programming skills (Python, Mathematica, etc). If you do not feel comfortable with these concepts, please come talk to me.

All exercises are from second edition of Jim Sethna's book available at:

`https://www.lassp.cornell.edu/sethna/StatMech/index.html`.
There are also hints and code for computational exercises at this website.

Readings and Pre-class questions

Tuesday:
Read: Chapter 6- Sethna
In-class question: Problem 6.6 Lagrange; MaxEnt.

Wednesday:
Pre-class question: 6.16 Rubber-band revisited
In class: Problem 6.4

Thursday:
Read Chapter Chapter 6 Sethna
In-class: Problems 6.7, 6.8,6.9

Tuesday:
Read Chapter Chapter 6 Sethna
In class: TBD

Wednesday:
Read Chapter Chapter 8.1-8.2 Sethna
Pre-class question: 8.3 Coin flips and Markov
In-class: 8.1 Ising Model (Please bring something that can run simulation at https://mattbierbaum.github.io/ising.js/ to class)

Thursday:
Read: Chapter 8 Sethna

In-class question: 8.5 Detailed balance; 8.20 Unicycle

Homework Exercises

Review Problems:

Please do following from Gould available here https://www.compadre.org/STP/.

1. Gould 4.38
2. Goud 5.32
3. Gould 5.33

Harder Problems:

All hints are available at: https://www.lassp.cornell.edu/sethna/StatMech/EOPCHintsAndMaterials.html or ask me directly.

4. Sethna 6.3 Negative Temperature
5. Sethna 6.24 Zipf's Law
6. **Freely Jointed-Chain (FJC) Model of a Polymer**: Previously, we modeled a polymer as a 1-dimensional random walker that could point in two directions $\pm\hat{\mathbf{z}}$ whose internal energy did not depend on direction.

We will now generalize this to three dimensions using partition functions. Let us a consider a polymer with N segments subject to an external force with magnitude F in the $\hat{\mathbf{z}}$ direction. We further assume that our polymer is immersed in a bath with temperature T and that the internal energy U_{int} of a segment does not depend on its orientation.

The net end-to-end displacement $\mathbf{r}$ will point along $\hat{\mathbf{z}}$. The length of the polymer L is given by $|\mathbf{r}|$, with $L \leq Nd$ where d is the length of a single segment of the polymer. Let us denote the direction of the i-th segment by the unit tangent vector $\hat{\mathbf{t}}_i$.

To write the partition function, we have to think about the energy E that we need to put into partition function.

(a) Argue that the correct partition function is

$$P(\{\hat{\mathbf{t}}_i\}) = \frac{e^{\frac{Fd}{k_B T}\sum_i \hat{\mathbf{z}}\cdot\hat{\mathbf{t}}_i}}{Z(T,F)} \tag{1}$$

(b) Calculate the partition function $Z(T,F)$ and use it to calculate the free energy $A(T,F)$ as function of the temperature and applied force.

(c) Calculate the average length $\langle L \rangle$ as function of T and F.

(d) What are the limiting expressions for high force and low force? Can you relate this to Hooke's law like we did in the simpler model?

(e) Calculate the average energy of the polymer and the entropy of the polymer.

Honor Code

All students are expected to follow the BU Honor Code. While collaboration is allowed and encouraged on HWs, each student should write up their own solutions. Copying HW is strictly forbidden. The students are allowed to consult all resources and books. However, students are NOT allowed to consult problem solutions from previous years or as found on the web.