PHYSICS

Thermalization in small quantum systems

A small closed quantum many-body system shows evidence of thermalization

By **Anatoli Polkovnikov** and **Dries Sels**

Chaos and ergodicity are the cornerstones of statistical physics and thermodynamics. Although classically, even small systems such as a particle in a two-dimensional cavity can exhibit chaotic behavior and thereby relax to a microcanonical ensemble, quantum systems formally cannot. However, recent theoretical work and, in particular, the eigenstate thermalization hypothesis (ETH), indicate that quantum systems can also thermalize. Indeed, ETH provides a framework connecting microscopic models and macroscopic phenomena, based on the notion of highly entangled quantum states. On page 794 of this issue, Kaufman *et al.* (*1*) demonstrate such thermalization in the relaxation dynamics of a small lattice system of interacting bosonic particles. By directly measuring the entanglement entropy of subsystems, as well as other observables, they show that after the initial transient time, the system locally relaxes to a thermal ensemble while globally maintaining a zero-entropy pure state.

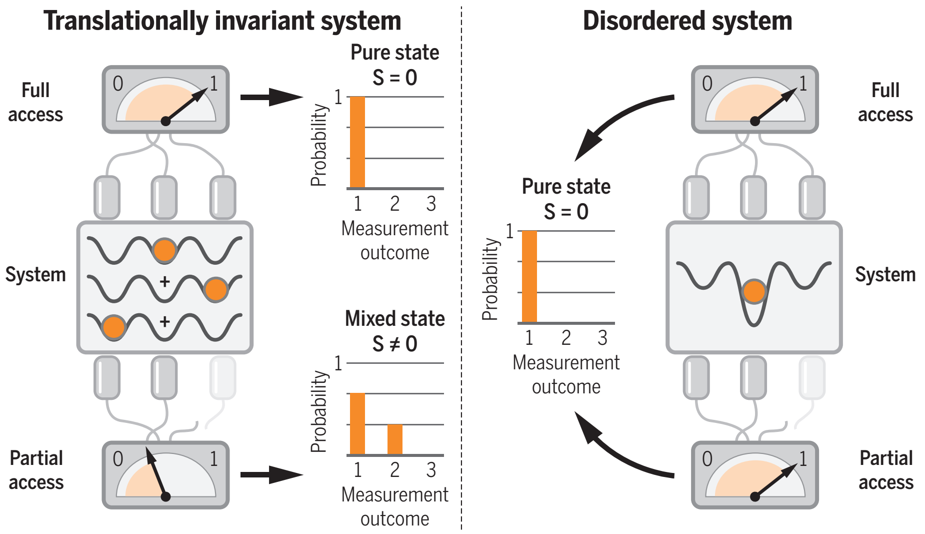
Department of Physics, Boston University, Boston, MA 02215, USA. Email: asp@buphy.bu.edu

The laws of thermodynamics are fundamental in nature as they do not rely on any specific microscopic theory. In particular, the second law postulates that any isolated system will reach an equilibrium state characterized by the maximum entropy under given macroscopic constraints such as total energy, number of particles, and volume. Apart from these constraints, all memory of the initial state is lost. This law is intrinsically irreversible as once a higher-entropy state is reached, there is no way to go back. At the same time, microscopic laws of nature are reversible. This apparent inconsistency has been a topic of controversy for over a century. In classical systems, it was partially resolved through chaotic motion occurring in generic nonlinear systems, which implies that after a transient time, any initial configuration reaches a typical state, on average occupying available phase space points with equal probabilities. However, there are still open questions: What is this transient time, and which configurations are typical?

Quantum mechanically, the situation looks even more confusing as the Schrödinger equation, which governs time evolution of the system, is linear and conserves the probabilities of occupying stationary (eigen-) states (*1, 2*). Therefore, the density matrix cannot relax to any statistical ensemble, but always retains information about the initial state even when allowing for time averaging. As a way out of this puzzling observation, von Neumann formulated ideas of typicality (*3, 4*), where quantum complexity of macroscopic systems should be hidden in an exponentially large number of quantum states. For example, the number of states describing a small $7 \times 7 \times 7$ system of spins is 2^{343}, far exceeding the number of particles in the universe. The physical information extractable from various observables is far less than what can be encoded in these many states. Consequently, most states have to be physically indistinguishable.

This observation, together with random matrix theory, developed and later applied to quantum chaotic systems (*5*), laid the foundations for the solution. In the 1990s the role of quantum chaos in the emergence of statistical mechanics was finally understood (*6, 7*), and the ETH was formulated. The ETH states that a single quantum eigenstate is equivalent to a microcanonical ensemble, in that they make identical predictions about physical observables. These ideas largely went unnoticed and were initially met with skepticism until they were confirmed numerically (*2*). The ETH elucidates the connections between microscopic laws and macroscopic phenomena, allowing precise and verifiable predictions to be made about chaotic systems (*8*). Yet ETH is only a conjecture, and there is no known rigorous way to derive it from first principles.

One of the most striking implications of ETH is that thermalization can occur even in relatively small systems. It is only sufficient to have a large Hilbert space, which scales exponentially with the system size. Thus, in the system experimentally realized by Kaufman *et al.* that consists of only six bosons confined to six lattice sites, the Hilbert space is already 462-dimensional. This



Entanglement in small quantum systems. (**Left**) An eigenstate of a particle in a translationally invariant system is a coherent superposition of three localized orbitals. An observer having access to the whole system can make a deterministic measurement giving reproducible identical results in each experimental realization. Conversely, an observer who can access only sites one and two can only see a statistical mixture of a particle in the state $(|01\rangle + |10\rangle)/\sqrt{2}$ with probability ⅔ and the state with no particles $|00\rangle$ with probability ⅓, and cannot make any deterministic measurement. According to ETH, in large chaotic systems, the reduced density matrices of stationary states describing small subsystems are maximally mixed, exactly as in a thermal ensemble. The level of mixing is encoded in the entanglement Rényi entropy S measured by Kaufman *et al.* (**Right**) In disordered systems, stationary states are localized; an observer having access to only a subsystem can still identify pure states with low entanglement. The many-body localization phenomenon (*12, 13*) implies that, surprisingly, such nonthermal states can be robust against interactions, preventing thermalization even in macroscopic systems.

GRAPHIC: C. BICKEL/SCIENCE

Published by AAAS

large dimensionality allows them to directly verify ETH predictions experimentally. Specifically, Kaufman *et al.* prepare two copies of the same system, with exactly one boson on every site. After a quantum quench, which allows particles to hop, correlations grow and the system becomes entangled. By performing a many-body interference experiment on the two copies, as suggested in (*9*) and tested experimentally in (*10*), the entanglement entropy of different subsystems as well as the entropy of the full state was measured (see the figure). Although the system as a whole remains pure, small subsystems are found to become mixed after a short transient time. Indeed, the reduced density matrices of one- and two-site subsystems become indistinguishable from those of a thermal ensemble. This equivalence is verified by direct observation of the particle occupation distribution and by comparing it with the equilibrium predictions. A recent experiment in a smaller system of three superconducting qubits (*11*) verified that the full time-averaged density matrix becomes thermal in chaotic regimes; another direct consequence of ETH (*8*).

Not only does ETH validate the use of statistical mechanics; there are also many important implications of these ideas to future science and technology. Understanding the microscopic structure of complex systems can provide the necessary tools and intuition for designing systems with similar or better performance than those found in nature, which often operate efficiently in far from ideal conditions. Understanding the conditions leading to the breakdown of ETH could be important for developing new technologies not suffering from the usual thermodynamic limitations. Remarkably, what first appeared to be an issue of controversy in quantum mechanics has provided an elegant solution to the problem of thermalization. It is the existence of individual highly entangled eigenstates that allows the somewhat ambiguous coarse-graining required in standard classical arguments to be dropped. Interestingly, ETH can be applied to systems near the classical limit, providing a simple mathematical framework to understand unanswered questions in classical chaotic systems. ■

REFERENCES

1. A. M. Kaufman *et al.*, *Science* **353**, 794 (2016).
2. M. Rigol, V. Dunjko, M. Olshanii, *Nature* **452**, 854 (2008).
3. J. Eisert, M. Friesdorf, C. Gogolin, *Nat. Phys.* **11**, 124 (2015).
4. S. Goldstein *et al.*, *Proc. R. Soc. A* **466**, 3203 (2010).
5. F. Borgonovi *et al.*, *Phys. Rep.* **626**, 1 (2016).
6. J. M. Deutsch, *Phys. Rev. A* **43**, 2046 (1991).
7. M. Srednicki, *Phys. Rev. E* **50**, 888 (1994).
8. L. D'Alessio *et al.*, *Adv. Phys.* **65**, 239 (2016).
9. A. J. Daley *et al.*, *Phys. Rev. Lett.* **109**, 020505 (2012).
10. R. Islam *et al.*, *Nature* **528**, 77 (2015).
11. C. Neill *et al.*; https://arxiv.org/abs/1601.00600 (2016).
12. M. Schreiber *et al.*, *Science* **349**, 842 (2015).
13. J.-Y. Choi *et al.*, *Science* **352**, 1547 (2016).

10.1126/science.aah5776

ECONOMICS

Fighting poverty with data

Machine learning algorithms measure and target poverty

By **Joshua Evan Blumenstock**

Policy-makers in the world's poorest countries are often forced to make decisions based on limited data. Consider Angola, which recently conducted its first postcolonial census. In the 44 years that elapsed between the prior census and the recent one, the country's population grew from 5.6 million to 24.3 million, and the country experienced a protracted civil war that displaced millions of citizens. In situations where reliable survey data are missing or out of date, a novel line of research offers promising alternatives. On page 790 of this issue, Jean *et al.* (*1*) apply recent advances in machine learning to high-resolution satellite imagery to accurately measure regional poverty in Africa.

Traditionally, wealth and poverty are measured through surveys of household income and consumption (*2*). These data provide a critical input to the world's most prominent antipoverty programs, from basic cash transfer programs to multifaceted aid programs designed to target the extreme poor (*3*). However, nationally representative surveys cost tens to hundreds of millions of dollars to collect, and many developing countries go for decades without updating their estimates.

"...there is exciting potential for adapting machine learning to fight poverty."

Over the past few decades, researchers have begun to develop different techniques for estimating poverty remotely. Initial work explored the potential of "nightlights" data: satellite photographs taken at night that capture light emitted from Earth's surface. Since such imagery first became available in the early 1970s, it was evident that wealthy regions tended to shine brightest (*4*). Recent studies have found a strong correlation between nightlight luminosity and traditional measures of economic productivity and growth (*5*, *6*). Nightlight-based measures are now frequently used by researchers, for instance to study the impact of sanctions on the economy of North Korea (*7*), where official statistics are dubious.

School of Information, University of California, Berkeley, CA 94720, USA. Email: jblumenstock@berkeley.edu

A series of studies in wealthy nations explore how data from the internet and social media can provide proxies for economic activity (*8*, *9*). Mining the tweets and search queries of millions of individuals promises real-time alternatives to more traditional methods of data collection. However, these approaches are less relevant to remote and developing regions, where internet infrastructure is limited and few people use social media.

In developing countries, researchers have found ways to measure wealth and poverty using the digital footprints left behind in the transaction logs of mobile phones, which are increasingly ubiquitous even in very poor regions. Regional patterns of mobile phone use correlate with the regional distribution of wealth (*10*). This relationship persists at the individual level, such that machine learning algorithms can infer an individual subscriber's socioeconomic status directly from his or her history of mobile phone use. The individual predictions can be aggregated into regional measures of wealth that are about as accurate as a 5-year-old household survey (*11*). Phone-based proxies for wealth are beginning to be used in research, e.g., to understand how new technologies differentially benefit the wealthy and the poor (*12*) and to assess the creditworthiness of would-be borrowers (*13*).

Although promising, these nontraditional methods have caveats. As Jean *et al.* show, nightlights data are less effective at differentiating between regions at the bottom end of the income distribution, where satellite images appear uniformly dark. And mobile phone data are owned by mobile phone operators and are generally not available to policy-makers. By contrast, Jean *et al.* use only publicly available data.

Taking nightlights as their starting point, the authors have devised a clever technique to also extract information from daytime satellite imagery. Daytime imagery is taken at much higher resolution than nighttime imagery. It thus contains visible features—such as paved roads and metal roofs—that make it possible to differentiate between poor and ultrapoor regions. Jean *et al.*'s insight was to apply state-of-the-art deep learning algorithms to the daytime imagery to extract these features. When given large quantities of data with labeled patterns, these algorithms

Thermalization in small quantum systems

Anatoli Polkovnikov and Dries Sels (August 18, 2016)

Science **353** (6301), 752-753. [doi: 10.1126/science.aah5776]