Homework 7 **PY501** (Fall 2012)

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Reading: This weeks HW is about using separation of variables to solve PDEs.

We focus on spherical coordinates . The lectures are from Mathews + Walker and Arfken Chapter 8+ Jackson.

Problems: Due Wednesday, December 12

1. Two concentric spheres have radii a, b ($b > a$) and each is divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the lower hemisphere of the outer sphere are maintained at potential V. The other hemispheres are at zero potential.

Determine the potential in the region $a \leq r \leq b$ as a series in Legendre polynomials. Include terms at least up to $l = 4$. Check you solution against known results in the limiting case $b \to \infty$ and $a \to 0$.

2. A spherical surface of radius R has charge uniformly distributed over its sphere with density $\frac{Q}{4\pi R^2}$, except for a spherical cap at the north pole, defined by the cone $\theta = \alpha$.

(a) Show that the potential inside the spherical surface can be expressed as

$$\Phi = \frac{Q}{8\pi\epsilon_0} \sum_{l=0}^{\infty} \frac{1}{2l+1} [P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)] \frac{r^l}{R^{l+1}} P_l(\cos\theta) \qquad (1)$$

(Hint: Notice that by Gauss's law the difference in electric fields across region with surface density must be related to surface charge density. This provides a boundary condition.)

(b) Find the magnitude and direction of the field at the origin.

(c) Discuss the limiting form of the potential (part a) and electric field (part b) as the spherical cap becomes (1) very small, and (2) so large that the area with charge on it becomes a very small cap at the south pole.

3. Three point charges (q, -2q, q) are located in a straight line with separation a and with the middle charge (-2q) at the origin of a grounded conducting spherical shell of radius $b > a$.

(a) Write down the potential of the three charges in the absence of the grounded sphere. Find the limiting form of the potential as $a \to 0$, but the product $qa^2 = Q$ remains finite. Write this latter answer in spherical coordinates.

(b) The presence of the grounded sphere of radius b alters the potential for $r < b$. The added potential can be viewed as caused by the surface-charge density induced on the inner surface at $r = b$ or by charges located at $r > b$. Use linear superposition to satisfy the boundary condition and the find the potential everywhere inside the sphere for $r < a$ and $r > a$. Show that in the limit $a \to 0$,

$$\Phi(r,\theta,\phi) \to \frac{Q}{2\pi\epsilon_0 r^3}(1 - \frac{r^5}{b^5})P_2(\cos\theta) \tag{2}$$

4. A sphere of radius R is at temperature $T = 0$ throughout. At time $t = 0$, it is immersed in a liquid bath at temperature T_0. Find the subsequent temperature distribution $T(r,t)$ inside the sphere. [Let κ= thermal conductivity/(density x specific heat).]

5. Find the lowest frequency of oscillation of acoustic wave in a hollow sphere of radius R. The boundary condition is $\frac{\partial\psi}{\partial r} = 0$ at $r = R$ and ψ obeys the differential equation

$$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \tag{3}$$

6. Calculate the Green's Function for the 1-Dimensional Diffusion Equation in the positive real axis, $x \geq 0$, when a random walker is placed at $x = x'$ at time $t = 0$. Use the boundary condition that there is a reflecting barrier at $x = 0$. In other words, random walkers hit $x = 0$ with some speed v are reflected with same speed. (Hint: Use the method of mirror charges).