Homework 6 PY501 (Fall 2012)
Instructor: Pankaj Mehta (323 SCI, pankajm@bu.edu)
Reading: This weeks HW is about using separation of variables to solve PDEs.

We focus on cylindrical coordinates . The lectures are from Mathews + Walker and Arfken Chapter 8+ Jackson.

Problems: Due Monday, December 2

1. Consider a rectangular box with dimensions (a, b, c) in the (x, y, z) directions positioned so that the box lies in the positive quadrant $x, y, z > 0$. All surfaces of the box are kept at zero potential everywhere inside the box except at $z = c$ which is kept at a potential $V(x, y)$. Solve Laplace's equation using separation of variables and find the potential Φ inside the box. Express your answer as a power series. (Hint: Expand the potential $V(x, y)$ as a Fourier series in x and y to get the coefficients of the series.)

2. (a) Two long half-cylinders of inner radius b are separated by a small lengthwise gaps on each side and kept at different potentials V_1 and V_2. Using separation of variables to solve for the potential and show that the potential inside is given by

$$\Phi(\rho, \phi) = \frac{V_1 + V_2}{2} + \frac{V_1 - V_2}{\pi} \tan^{-1} \frac{2b\rho}{b^2 - \rho^2} \cos\phi. \quad (1)$$

where ϕ is measured perpendicular to the plane through the gap.

(b) Calculate the surface charge density on each half of the cylinder.

3. A hollow right circular cylinder of radius b has its axis coincident with the z-axis and its ends at $z = 0$ and $z = L$. The potential at the end faces is zero, while the potential on the cylindrical surface is given by $V(\phi, z)$. Use the appropriate separation of variables in cylindrical coordinates, find a series solution for the potential anywhere inside the cylinder. (Hint: Think about what the boundary conditions on the ends of the cylinders imply about the choice of parameters in the separation of variables and Bessel equation).

4. An infinite, thin plane sheet of conducting material has a circular hole of radius a cut in it. A thin, flat disc of the same material and slightly smaller radius lies in the plane filling the hole, but separated from the sheet by a very narrow insulating ring. The disc is maintained at a fixed potential V, while the infinite sheet is kept at potential 0.
a) Using the appropriate cylindrical coordinates, find an integral expression

involving the Bessel function for the potential at any point above the plane.
b)Use the identity

$$\frac{1}{\sqrt{\rho^2 + z^2}} = \int_0^\infty e^{-k|z|} J_0(k\rho) \tag{2}$$

to show that the potential a perpendicular distance z above center of the disc is

$$\Phi_0(z) = V(1 - \frac{z}{\sqrt{a^2 + z^2}}) \tag{3}$$